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**GRAPHICAL SOLUTIONS
OF
GEOLOGIC PROBLEMS**

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OF
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GRAPHICAL SOLUTIONS OF GEOLOGIC PROBLEMS

Introduction

The geologists and engineers in the Soil Conservation Service in the course of their duties encounter problems in determining the true location, attitude, or orientation of geologic structures.

These problems can usually be solved mathematically but the mathematics is often quite involved. Graphical methods that are rapid and accurate to use will give valid results.

Scope

This technical release covers some of the basic techniques in graphical solutions of three dimensional problems involving points, lines, and planes. These techniques will give attitude, location, distance, and dimensions in the solution. The techniques of using hemispherical nets for the solution of problems are also covered. These techniques while they give attitudes and direction do not provide distance and dimension in the solutions.

The procedures presented herein are not new or original but have been used by geologists for a number of years. This is a compilation that has assembled material from various sources into one document that will be readily accessible to SCS geologists. If additional information is desired about these techniques the reader is referred to the references listed at the end of this TR.

Orthographic Projections

A geologic structure has a fixed position in the earth's crust. When this position has been determined by a survey method (transit, plane table and alidade, or compass bearing) the observer must consider this position as fixed. If the observer wishes to view this structure from another position, he must look directly at the position he wishes to see. In a sense, the observer must think and visualize the structure in three dimensions. If he does this, he can always observe directly the view he wishes to see.

The orthographic projection is a right-angle type of projection. It uses parallel lines for projection at right angles to an image plane. The image plane is the plane on which a view is projected. A folding line is the intersection of two image planes.

Drawing equipment needed for solution of problems by orthographic projection are: paper, T-square and/or triangles, scale, protractor, and drawing pencils. Dividers and a compass are useful at times, but a scale can usually be used. The lines drawn should be fine and sharp and points well defined. All angles and measurements must be laid off accurately.

The plan view is the basic view. All other views must be rotated about folding lines into the image plane which is the plane of the paper. Figure 1 is an illustration of several views projected by orthographic projection. Basic information given is: *a stratum of rock outcrops, the width of the outcrop is 0, and the stratum dips 45° due south.* The isometric sketch (not to scale) in the lower right-hand corner is the block of rock we are considering in this figure.

From the given information views 1 and 2 are constructed. The plan view (View 1) is drawn first. The view that can be drawn next is a north-south cross section. This cross section is rotated into the plane of the paper by rotation around a folding line (FL) drawn in a north-south direction. The points A1, B1,...H1, are projected perpendicular to the folding line to view 2. Point C2F2 is located any convenient distance below the folding line, the 45° angle laid off, and a line drawn from C2F2 to the intersection of the line projecting points A1B1 from view 1 to view 2. Line E2D2-A2G2 is parallel to C2F2-B2H2 and the various points found at the intersection of projection lines from view 1.

It is desirable to label all points and folding lines. In Figure 1 the points are all labeled with a letter and number. The letter designation remains the same in all views while the number portion changes to the view number. A convenient method of labeling folding lines is by use of the symbol FL to indicate a folding line and a two-number designation showing the view projected from and the view projected to. In Figure 1 in the label FL 1/2, FL indicates the folding line; the 1 indicates on which side of the folding line view 1 is located and that it was drawn first; and the 2 indicates which side of the folding line view 2 is located and that it was drawn second and by projection from view 1.

View 3 was drawn third by projection from view 1 as indicated by the notation FL 1/3 on the folding line. View 3 is related to view 1 in the same manner as view 2 is related to view 1. Therefore, all points in view 3 (A3, B3,...H3) are located the same distance from FL 1/3 as points A2, B2,...H2 are from FL 1/2 in view 2.

View 5 was constructed by projection from view 4 perpendicular to FL 4/5. View 4 is related to view 5 and to view 1, therefore, point A5 is the same distance from FL 4/5 as point A1 is from FL 4/1.

This relationship of views is the basis of orthographic projection. Enough information to construct 2 views must be available if additional views are to be constructed.

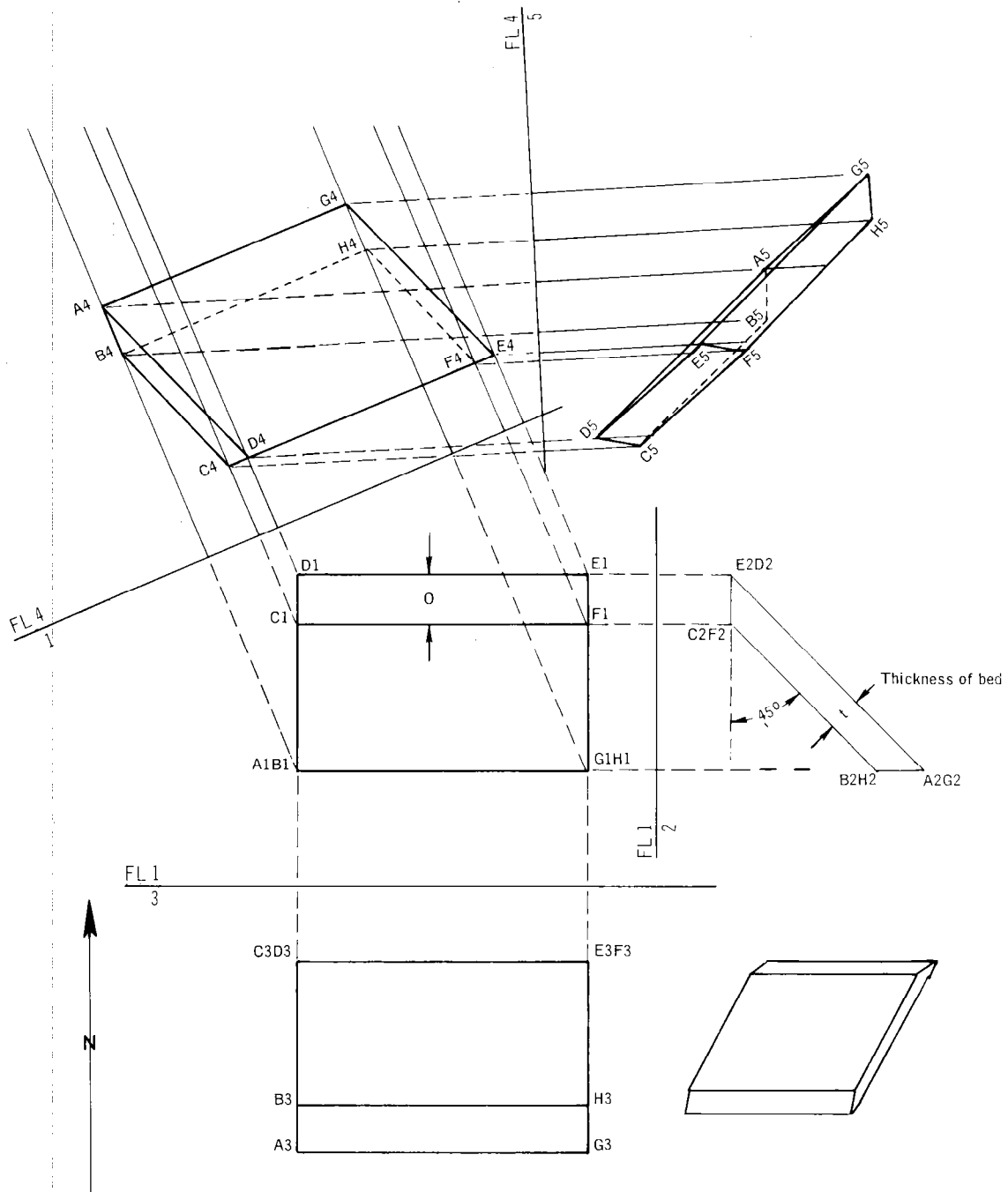


FIGURE 1.—Orthographic projection.

Depth to a Dipping Bed

Depth to a dipping bed may be readily determined if the dip and strike of the bed and surface elevations are known.

Figure 2 is the graphical solution of a problem with the following data known: At point A the top of a shale bed with a dip of 15° to the S45°W outcrops; at point B 100 feet due east of A the bottom of the shale bed outcrops with the same dip and strike; across a ridge 292 feet due west is the low point in a valley on the centerline of a structure. Assume all three points are the same elevation. At what depth would the top of the shale be encountered in a test hole, what thickness of shale would be penetrated by a vertical test hole, what is the true thickness of the shale, and what is the outcrop width of the shale?

To solve this problem, points A and B and the test hole are located on a plan view. The strike of the shale at points A and B is drawn (N45°W) and the direction of the dip indicated. A folding line (FL 1/2) is drawn east of point B and the strike of the two beds is projected to the folding line. An angle of 15° is laid off between the FL and point A (or B) and the top and bottom of the shale bed is drawn. The test hole is projected at right angles to FL 1/2 and the depth, thickness, etc., measured from the drawing. If the elevation of the test hole is different than points A and B, the difference in elevation can be subtracted or added to the depth (60 feet) as scaled from the drawing.

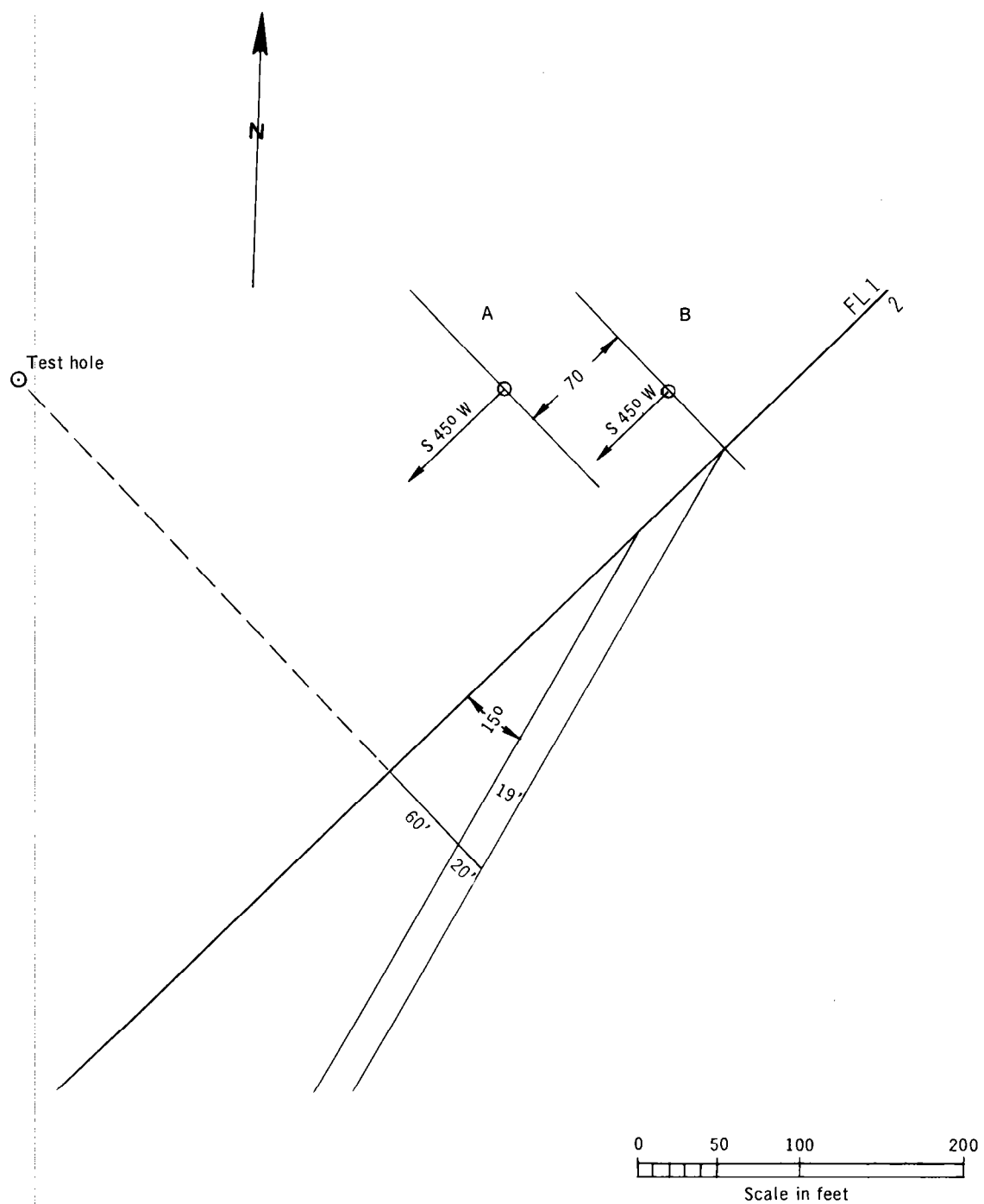


FIGURE 2.—Orthographic projection.

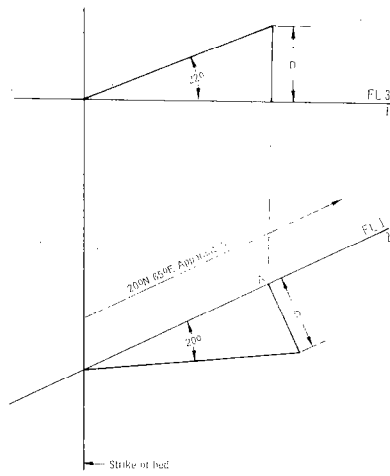
Determine True Dip from One Apparent Dip and the Strike

A bed strikes north-south and has an apparent dip of 20° to the $N65^\circ E$. What is the true dip of the bed?

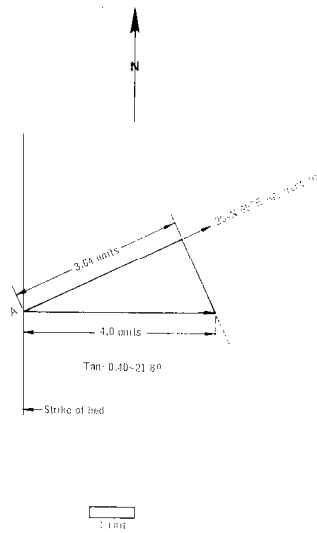
Figure 3 shows the solution of this problem by two methods--orthographic projection and tangent vector method.

In the orthographic projection (Figure 3-A) a plan view is drawn showing the strike and direction of apparent dip. The apparent dip is rotated into the plane of the paper around FL 1/2 and the apparent dip angle of 20° is laid off with a protractor. At any convenient distance along the folding line, such as point A, a perpendicular is dropped from FL 1/2 to the dipping bed and the distance D is measured. Folding line 3/1 is drawn at right angles to the strike of the bed and point A is projected FL 3/1 at right angles to FL 3/1. The distance D is measured in view 3, the bed drawn in and the angle of true dip (22°) measured with a protractor. The direction of dip is at right angles to the strike or due east.

The tangent vector method is used in figure 3-B to solve the problem. In this solution the strike and direction of apparent dip are plotted in the plan view. A table of trigonometric functions (or a slide rule) is used to obtain the value of the tangent of 20° (0.364). Along the apparent dip line in the plan view 3.64 units (a unit is any convenient length) are laid off. A perpendicular is dropped from the apparent dip line to the line representing the bearing of true dip and the distance measured from the intersection to point A (4.0 units in this case). The table of trigonometric functions or the slide rule is used to find the angle whose tangent is 0.40. The true dip is 21.8° due east.



A. Orthographic projection.



B. Tangent vector method

FIGURE 3.—True dip from apparent dip and strike

Determine True Dip from Two Apparent Dip Measurements at Same Point

Two apparent dips measured at point A are: $30^{\circ}\text{N}40^{\circ}\text{E}$ and $15^{\circ}\text{N}15^{\circ}\text{E}$. Find the angle and direction of true dip.

Figure 4 is a solution to this problem by orthographic projection. The two apparent dips originating at point A are plotted in the plan view. FL 2/1 and FL 1/3 parallel to the two apparent dips are drawn and the apparent dip angles (15° and 30°) are plotted.

At any convenient point on FL 2/1 a perpendicular is dropped to the dipping bed and the distance D measured. This point on FL 2/1 is projected to the bearing of the apparent dip (point 1) in the plan view. The point where a perpendicular with a length of D from FL 1/3 to the dipping bed is located on FL 1/3 and projected to the plan view as point 2. Points 1 and 2 in the plan are the location of points of the same elevation on the dipping bed. A line connecting points 1 and 2 is the true strike of the bed ($\text{N}4^{\circ}\text{W}$).

The true dip is perpendicular to the strike or $\text{N}86^{\circ}\text{E}$. The amount of true dip is found by laying off the same distance D in a view perpendicular to and along the same strike line defined by points 1 and 2. This is shown in view 4.

A less cluttered drawing for the solution of this problem can be constructed by using the lines indicating the direction of apparent dips and true dip as the folding lines. This is illustrated in Figure 5.

Figure 6 is a solution by the tangent vector method of the same problem. The plan view is drawn. The tangent of 15° is .268 and the tangent of 30° is .577. Therefore, 2.68 and 5.77 units are laid off along the respective apparent dip bearing lines. Perpendiculars are drawn. A true dip bearing line is drawn from point A to the intersection of the two perpendiculars and the distance (8.27 units) measured to give a true dip of $39.6^{\circ}\text{N}86.5^{\circ}\text{E}$.

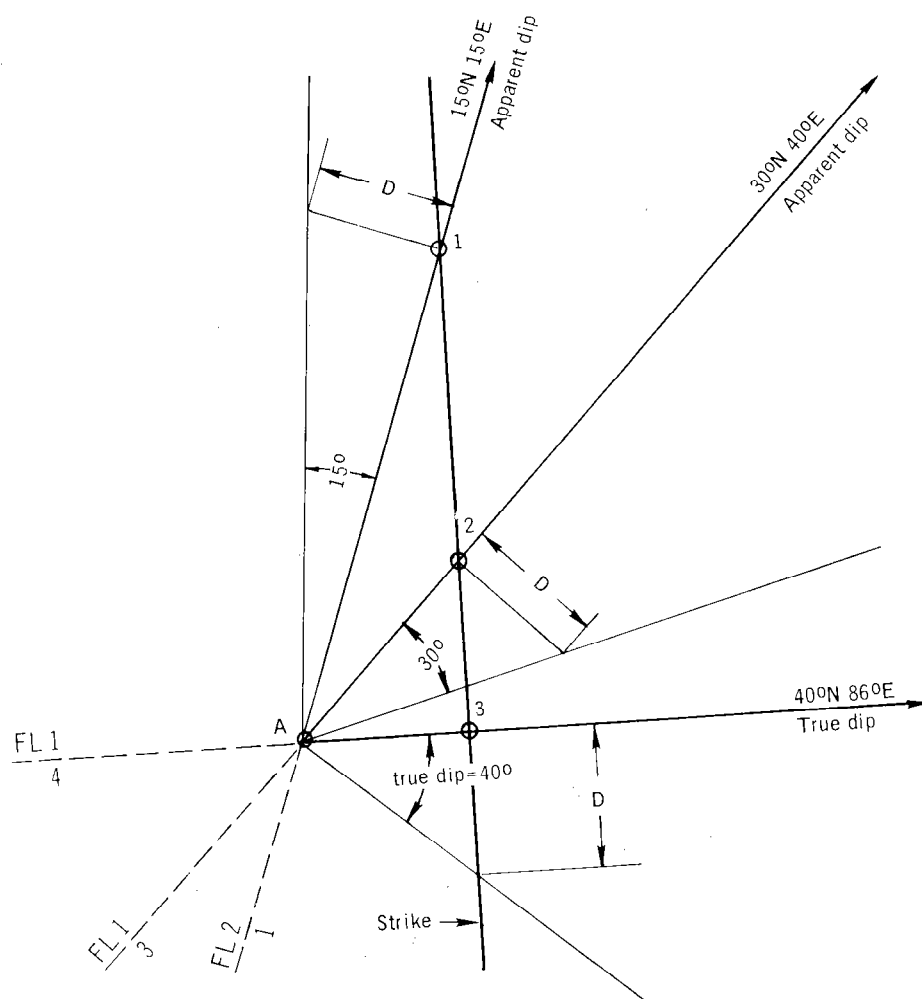


FIGURE 5.— True dip from two apparent dips.

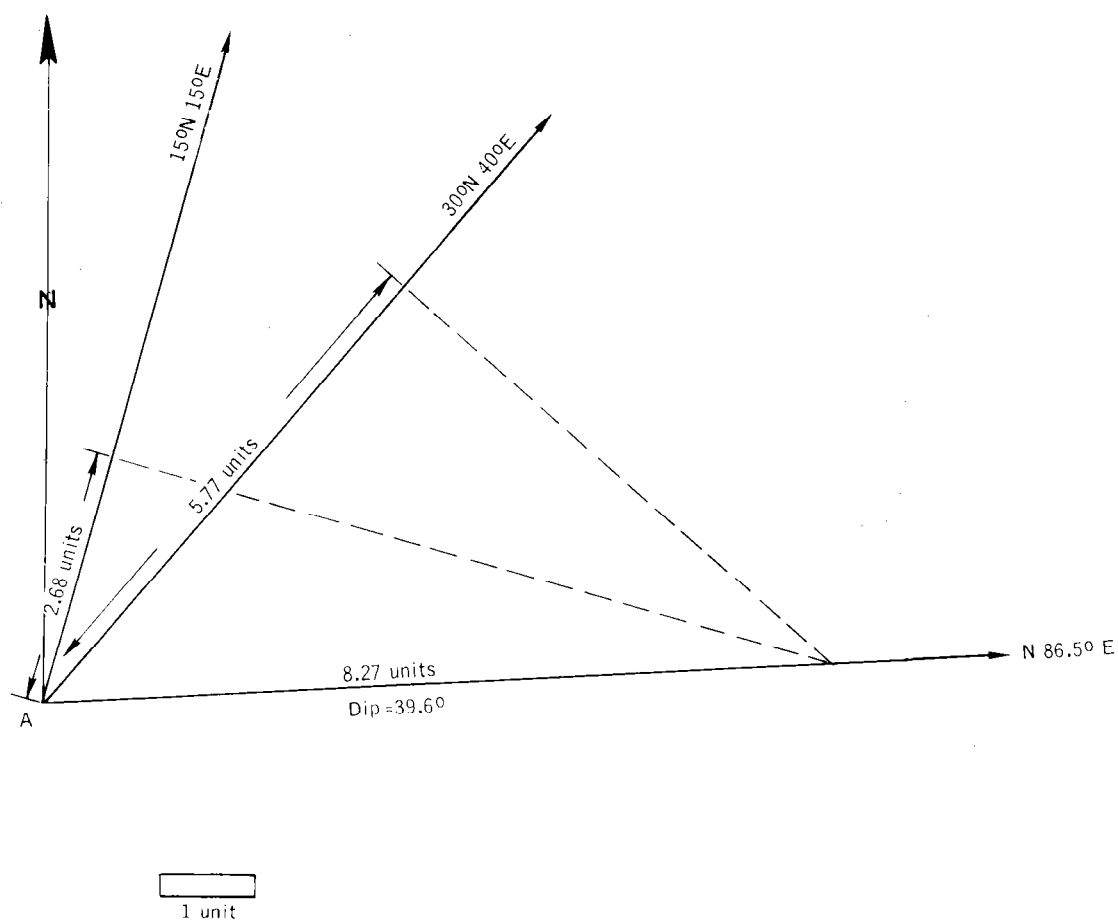


FIGURE 6.—Tangent vector method.

Three Point Problem

Any three points on a plane define the location of that plane if they are not in a straight line. Therefore, the dip and strike of a true plane surface can be determined from three points. An example of the solution of this type of problem follows.

In a test well at point A a key marker bed is encountered at an elevation of 850 feet. Point B, the second test well, is 1000 feet due west of point A and the marker bed is encountered at an elevation of 620 feet. At point C, 800 feet S25°E from point B, a third test well encounters this marker bed at elevation 720 feet. What is the true dip and strike of this marker bed?

The location of the three points are plotted at a convenient scale in the plan view (Figure 7-A). Line AE is drawn above AB (highest and lowest points) and distances equivalent to the difference in elevation between points A and B laid off at a convenient scale. A line is drawn from the 230 mark on AE to point B (difference in elevation between A and B) and a parallel line drawn from the 130 point to line AB. The intersection on AB is at an elevation of 720, the same as point C, and a line connecting this intersection and point C is the strike of the bed. FL 1/2 is drawn at right angles to the strike and points A, C, and B are projected perpendicular to the FL. Point A is on the FL (highest point), point C is 130 feet below, and point B is 230 feet below. A line drawn through these 3 points defines the true angle of dip in view 2.

Figure 7B is an alternate solution of the same problem. Sections from the highest point (A) to the other two points (B and C) can be considered as two apparent dips and the problem solved as described previously under two apparent dips from the same point. In the solution FL 1/2 and FL 3/1 are drawn and 130 feet laid off perpendicular to FL 1/2 at C and 230 feet perpendicular to FL 3/1 at B. Point D is the projection in the plan view where the bed is 130 feet below point A in view 3. Point C and point D are both 130 feet below A in the plan view and a line connecting these points defines the strike. FL 4/1 is drawn through point A and perpendicular to the strike and 130 feet scaled off in view 4 perpendicular to FL 4/1 along the strike line defined in the plan view. This is the angle of true dip; the bearing of true dip is at right angles to the strike.

In problems similar to the above if the dip angles are very small, it is difficult to measure them with a protractor. If these angles are converted to linear measurements (feet/mile, feet/feet, etc.) the vertical scale can be exaggerated (10, 100, etc., times) to provide a workable drawing. It is important to remember if an exaggerated vertical scale is used the dip angles cannot be measured with a protractor.

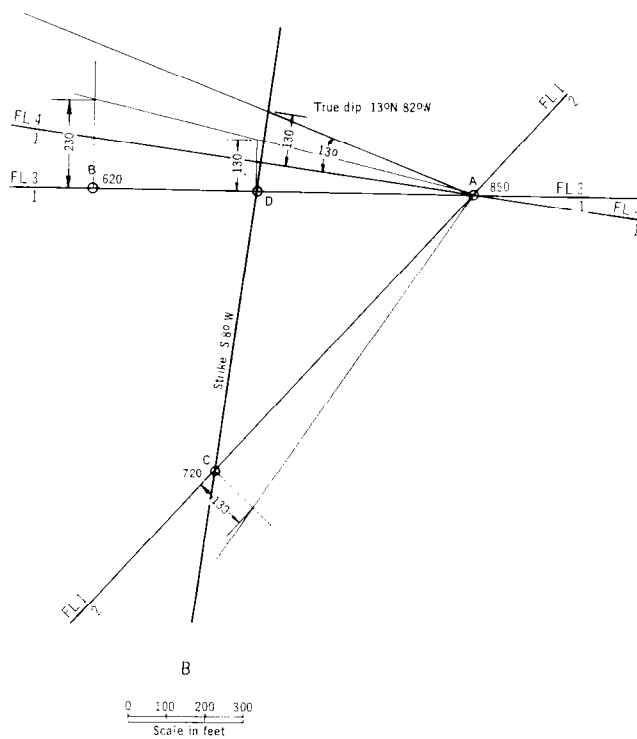
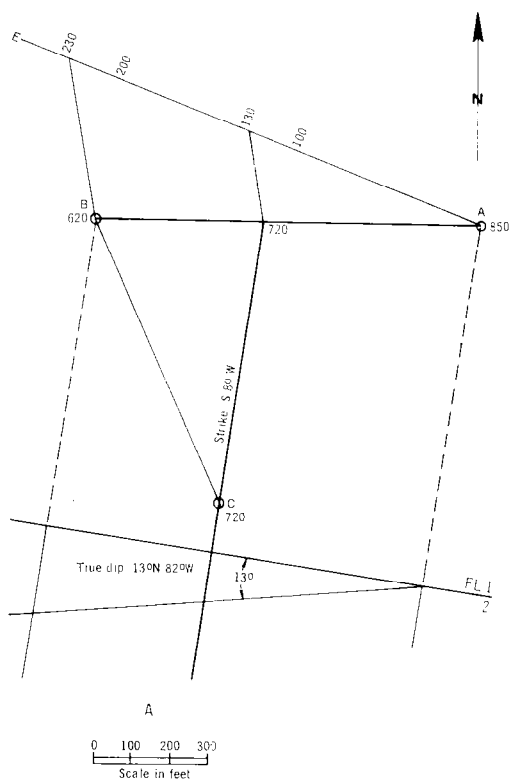


FIGURE 7 - Three point problem.

Problems Involving Points, Lines, and Planes

In working problems involving points, lines, and planes, a line is assumed to be straight throughout its course and a plane is assumed to be a true plane.

A line, to be shown in its true length in a view, must be projected to that view by lines of sight that are at right angles to the line in the first view. Stated in another way, to project a line to a view where it will be shown in its true length, the folding line (FL) is parallel to the line in the first view and the lines of sight are perpendicular to the FL.

The true slope of a line can be seen only in an elevation view which shows the line in its true length. The true slope can only be projected from the plan view. The true length of a line can be projected from views other than the plan view.

A line will appear as a point in a view taken at right angles to the line shown in its true length.

A plane will appear as a line in the view in which any line in the plane appears as a point. Therefore, the true direction and angle of dip of a plane will be shown in the elevation view at right angles to the strike.

Problems Involving Points and Lines

A line dips $20^{\circ}N20^{\circ}E$ and outcrops at point A. Point B lies 1000 feet $N60^{\circ}W$ from point A and is 200 feet lower. What is the distance and slope in a due east direction from point B to the line? What is the shortest distance, direction, and slope from point B to the line?

Figure 8 is the solution for the distance and slope in a due east direction. The plan view (view 1) is drawn from the given data. Note that line B1-C1 is in a due east direction. FL 1/2 is parallel to dipping line originating at A and the angle of dip (20°) is laid off. Point B2 in view 2 is 200 feet below A2 and point C2 is on the dipping line as projected from view 1. FL 3/1 is drawn parallel to B1-C1 and these points projected to view 3. The true length (1050 feet) and true slope ($+0.5^{\circ}$) of the line is found in view 3. Note: the distances from FL 3/1 to points B3 and C3 are equal to the distances from FL 1/2 to point B2 and C2.

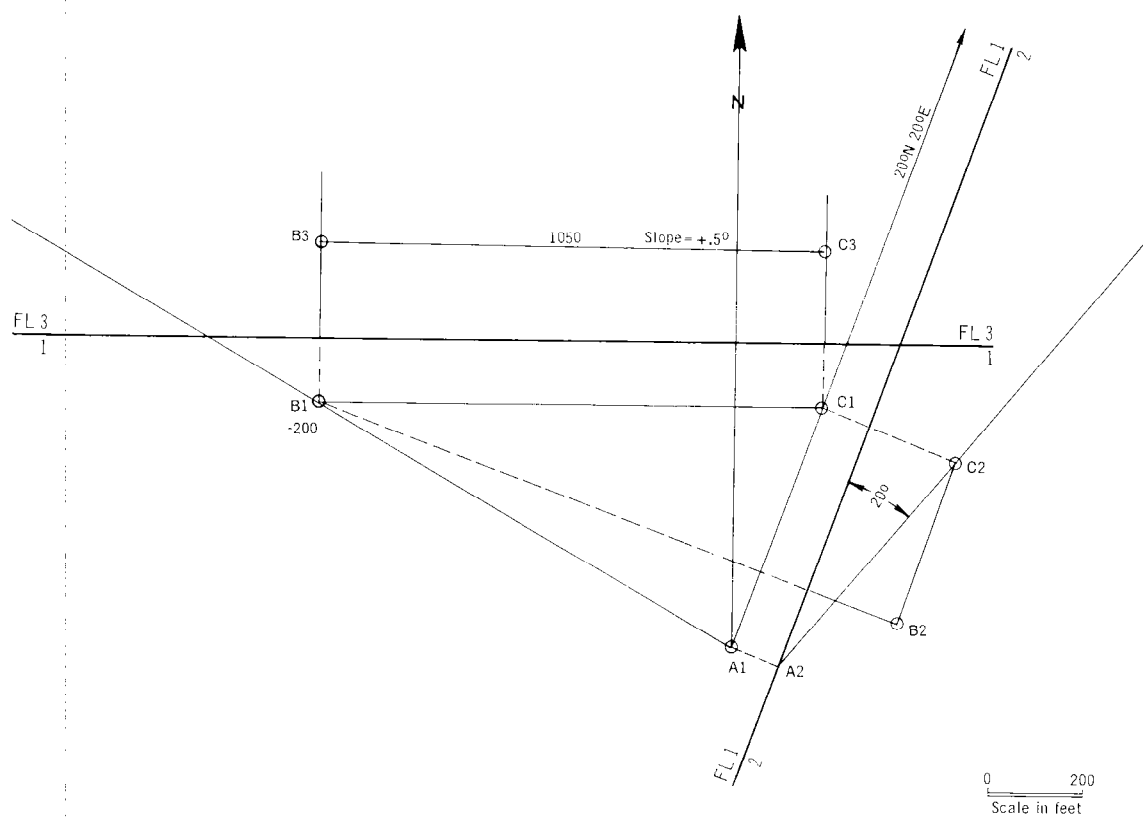


FIGURE 8. - Three point problem.

Figure 9 is the solution for the shortest distance from point B to the sloping line. The plan view is drawn except for point C1 which is unknown. View 2 is drawn. The shortest distance from point B to the sloping line is perpendicular to the sloping line in view 2. Perpendicular B2-C2 is drawn and point C projected to the plan view (view 1). Line B1-C1 is the true bearing of the shortest line, the true length and slope is found by projection to view 3.

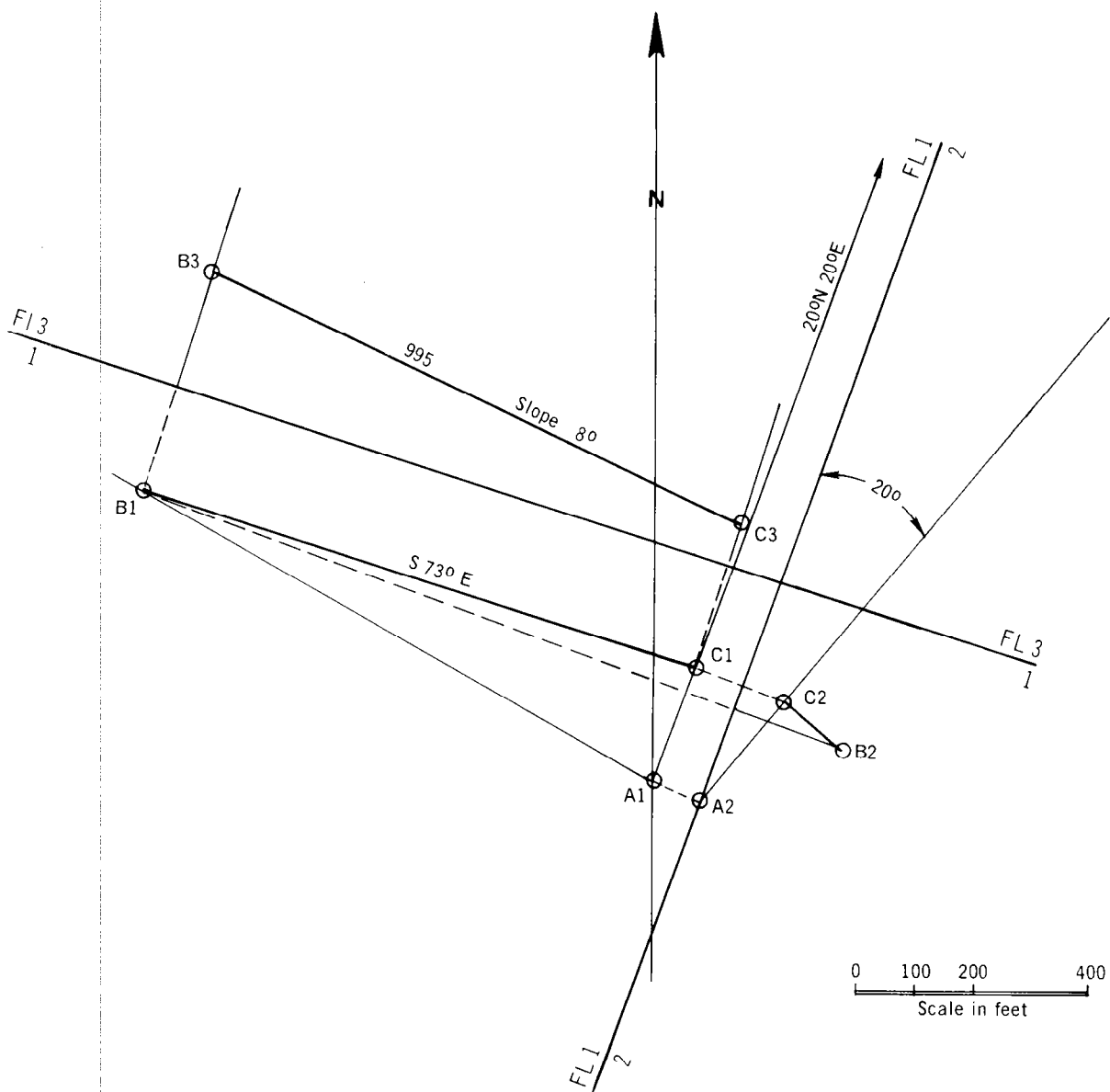


FIGURE 9.—Distance from a point to a line.

Shortest Distance Between Two Non-Parallel, Non-Intersecting Lines

An inclined line outcrops at point A and dips 20° true north. Another line outcrops at point C, 1500 feet $N45^\circ E$ from point A and 500 feet lower and dips $30^\circ W$ NW ($N67.5^\circ W$). Find the azimuth, slope, and length of the shortest line connecting these two inclined lines and the distance from point A and point C to the intersection of this connecting line.

The plan view showing the true azimuth of the two lines is drawn in view 1, Figure 10. Points B1 and D1 are arbitrary points plotted to provide two points on a line so the line may be projected to other views. View 3 is drawn to show the true length and slope of line CD. View 2 is drawn to show the true length and slope of AB, and CD is also projected to this view. View 4 (FL 2/4) is projected perpendicular to line A2-B2. In view 4, this line is shown as point A4-B4.

The shortest distance from a line to a point is perpendicular to the line. In view 4, X4-Y4 is perpendicular to C4-D4 and is the shortest distance between line C4-D4 and point A4B4. It is also shown in its true length (540 feet). Point X4 is projected back to view 2 and the shortest distance (X2-Y2) from a point (X2) to a line (A2-B2) is again perpendicular to the line. Line X2-Y2 is not, however, shown in its true length in this view. Points X2 and Y2 are projected to view 1 and line X1-Y1 is the true bearing ($S46^\circ E$) of the intersecting line connecting A1-B1 and C1-D1. The true slope of this intersecting line ($57\frac{1}{2}^\circ$) is shown by projecting to view 5 (FL 1/5 is parallel to X1-Y1) and the true length is again shown and checks (540 feet) with view 4. As a further check on the accuracy of the drawing, view 6 perpendicular to line C3-D3 could be made and same procedure of projections repeated.

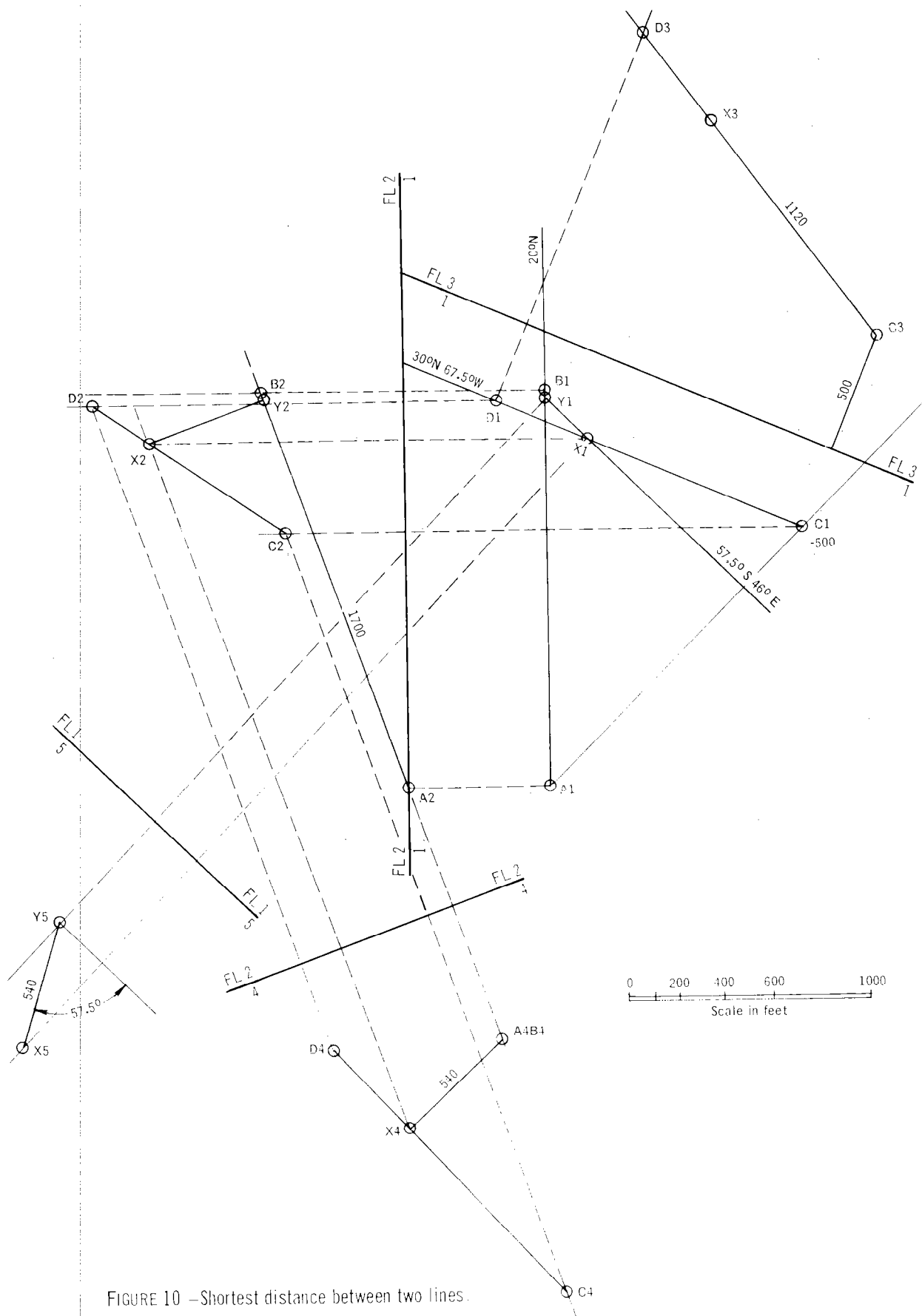


FIGURE 10 –Shortest distance between two lines.

Distance From a Point to a Plane

Points A, B, and C are three points on a plane. Point A has an elevation of 300 feet. Point B has an elevation of 500 feet and is 1500 feet due east of A. Point C has an elevation of 800 feet and is 1750 feet from point A and 1500 feet from point B. Point E is 300 feet N45°W of point A at an elevation of 300 feet. Find the distance from point E to the plane in the direction S45°E with a plunge of 20°. Locate the piercing point of the line and plane and the angle between the line and plane.

The location of the piercing point of the line and plane is found in an edge view of the plane. The distance from point E to the piercing point is found in the projection that shows a true view of the line. The angle between the line and plane is seen in the view that shows a true view of the line and an edge view of the plane.

Figure 11 is the solution of this problem. The plan view (view 1) and view 2 (S45°E) are drawn. The 20° angle is laid off in view 2 and arbitrary point F2 picked and projected to view 1. View 3 is any vertical section and drawn by projecting points A, B, and C and plotting them at their proper elevation. Points E and F are projected with measurements obtained from view 2. A level (strike) line B3-D3 parallel to F1 1/3 is drawn in view 3 and point D projected to the plan view. Line B1-D1 is the strike of the plane. To find the piercing point of the line in the plane, an edge view of the plane is needed. This is done in view 4 with FL 1/4 perpendicular to the strike (B1-D1) determined in view 1. Note that in view 4 the plane as defined by points A, B, and C must be extended to locate the piercing point (P4). The piercing point (P4) can be projected back to view 2 (through view 1) to show the length of EP (150 feet) in its true view.

The angle the line EP makes with the plane can only be seen in its true position in a true view of the line and an edge view of the plane. This requires two additional projections. First view 5 is drawn parallel to A4-C4-B4, to show the plane in a true view and then view 6 is drawn parallel to E5-P5-F5 to show the line in a true view and the plane as an edge view. The distance D6-P6, the true length, checks with distance E2-P2 (150 feet) and angle B6-P6-F6 can be measured (36°) which is the true angle between the line and plane.

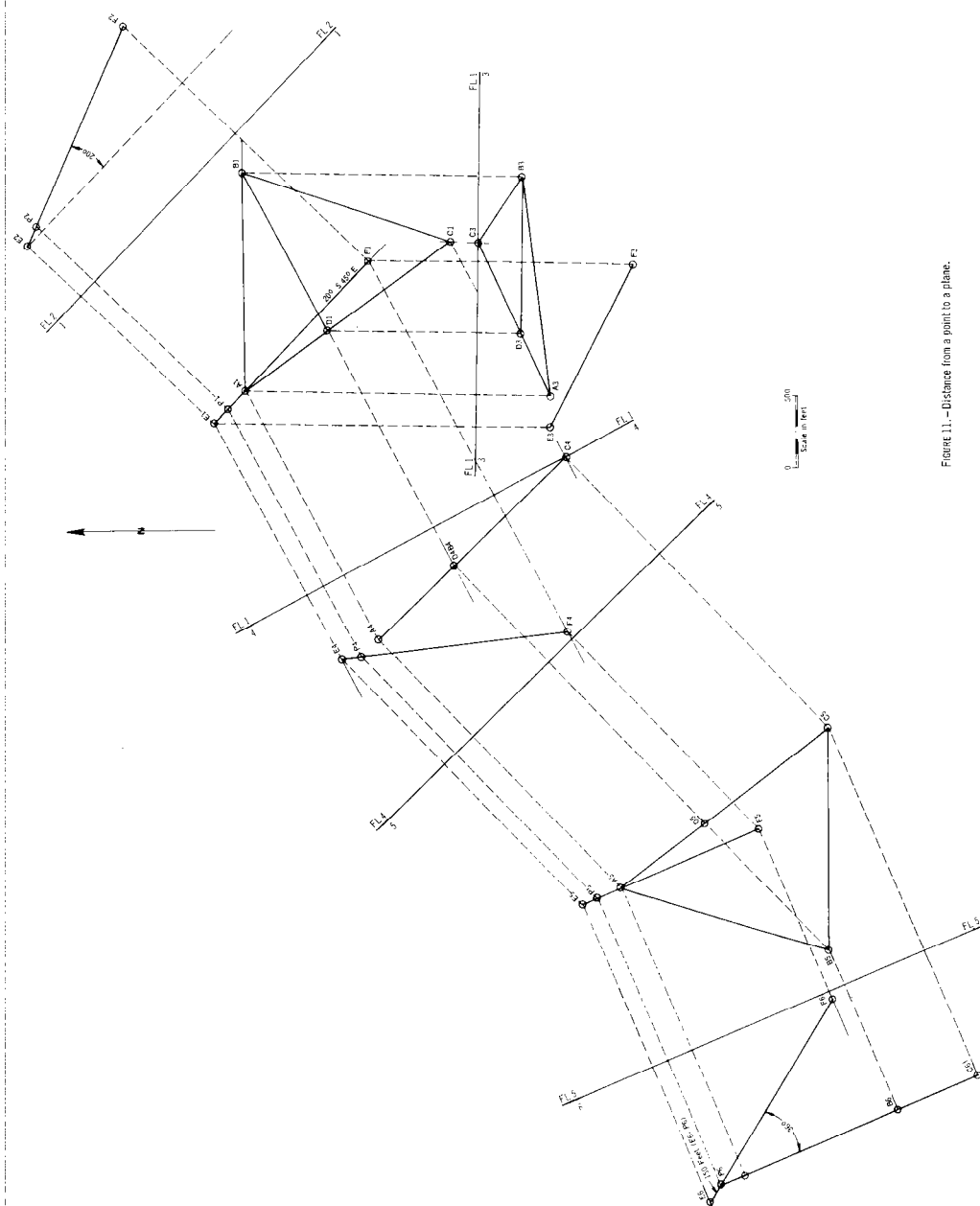
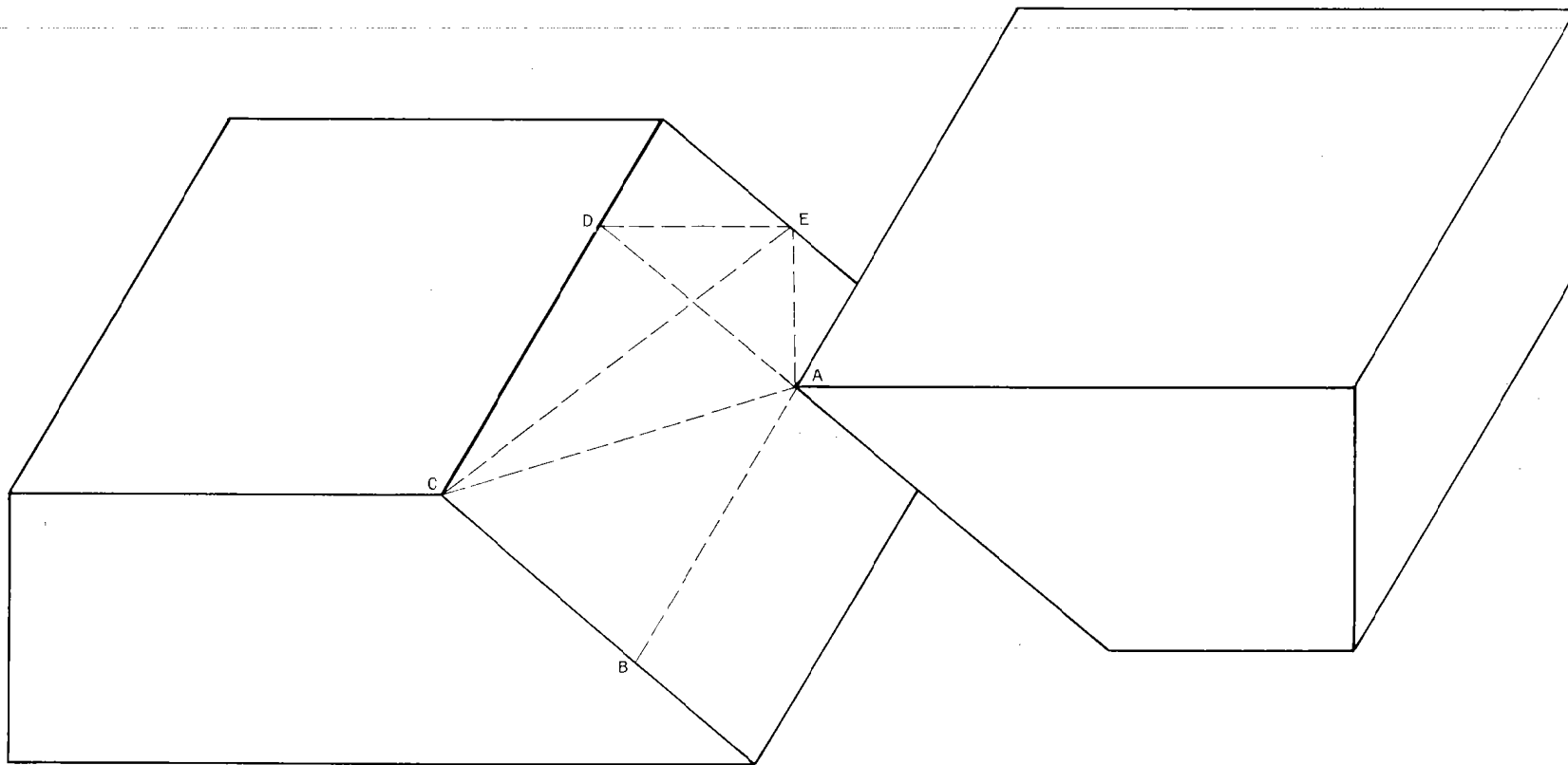


Figure 1.1.—Distance from a point to a plane.

The nomenclature for fault displacements as used in this technical release is illustrated in Figure 12. This is the same nomenclature as used by Billings (1954).



CA	net slip	ED	heave
DA	dip slip	Angle ACD	rake
BA	strike slip	Angle ECA	plunge
EA	throw		

FIGURE 12.—Nomenclature of fault displacement.

Determine the Line of Intersection of Two Oblique Planes

In the following example determine the bearing and plunge of the line of intersection between two planes and the rake (pitch) of that line in each plane. The given information is: a bed dips $30^{\circ}\text{N}10^{\circ}\text{W}$ and a fault dips $20^{\circ}\text{S}45^{\circ}\text{W}$.

In Figure 13 the plan view, view 1, is drawn from the given information. Draw the strike of the bed and fault and indicate the direction of dip. Next, views 2 and 3 are drawn perpendicular to the strike and the amount of dip is plotted for each. At an arbitrary distance X below and parallel to folding lines 2/1 and 1/3 an auxillary plane is drawn. This auxillary plane defines a common distance below the plan view and remains constant throughout the solution of the problem. Points A and B are the projections to the plan view of the intersection of the fault and the bed with the auxillary plane. These points (A and B) define, in the plan view, a point on the fault and the bed that is X distance beneath the surface. A line drawn from A to C parallel to the strike of the fault is a structure contour on the fault plane. The line from B to C is also a structure contour on the bed at the same elevation as the structure contour on the fault. These two structure contours intersect at point C. Point O is the intersection of the fault and bed in the plan view and point C is the projection into the plan view of the intersection of the auxillary plane or structure contours. Two points on the intersection determine the bearing of the intersection, therefore, line OC connecting these points is the bearing ($\text{N}79^{\circ}\text{W}$) of the intersection of the two planes. The plunge of the intersection is determined in a vertical section. FL 4/1 is drawn parallel to $\text{N}79^{\circ}\text{W}$, points O and C projected perpendicular to FL 4/1 and distance X laid off on the projection of point C and the angle measured.

To determine the rake (pitch) of the intersection in the plane of the fault, it is necessary to rotate the fault into the plan view. Use GH as a radius and G as the center, draw an arc to intersect FL 1/3. This point on FL 1/3 is the location of point H when the fault is rotated into a horizontal position. A line is drawn from this point on FL 1/3 parallel to the strike of the fault (this line is also a structure contour line X distance beneath the surface rotated to the surface). A perpendicular from this line to point C defines point D. This is the same relationship as point A has on the GH arc on FL 1/3. Point D is the projection of point C when the fault is rotated into the horizontal (plan) view. The rake of the intersection of these two planes in the plane of the fault is measured between the strike of the fault and line OD, which is on the plane of the fault rotated into the plan view.

The rake of the intersection in the plane of the bed is determined in the same manner starting from view 2 and rotating the auxillary plane into the horizontal.

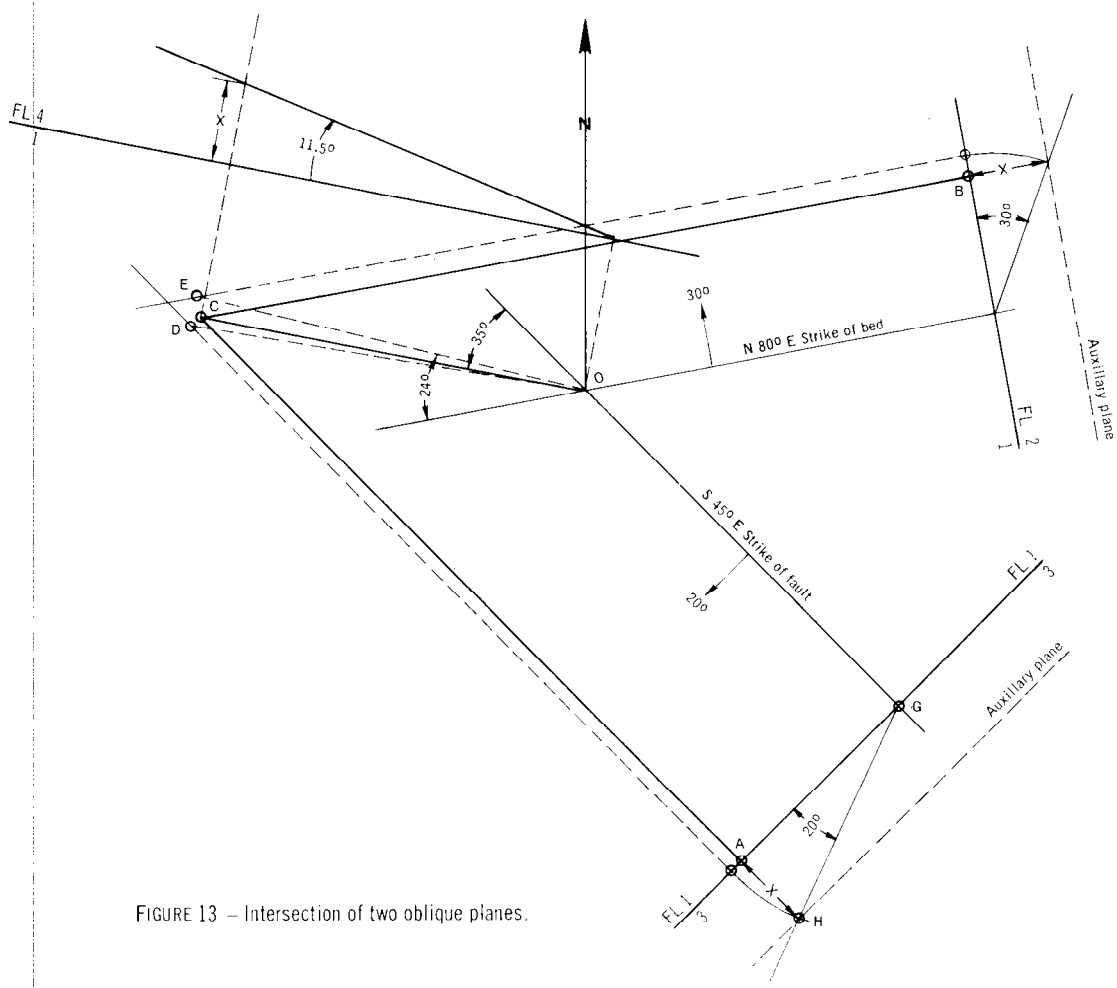


FIGURE 13 – Intersection of two oblique planes.

Displacement of a Vertical Fault

The displacement of a vertical fault can be determined if the attitude and location of two displaced horizons on each side of the fault are known. The location of additional horizons on one side of the fault can be found if their location on the other side is known.

Figure 14 is the graphical solution of the following problem: *A vertical fault strikes east-west and is exposed at point A; a vein with a dip of $30^{\circ}S30^{\circ}E$ outcrops at point B on the north side of the fault and point C on the south side of the fault. Another vein with a dip of $45^{\circ}S45^{\circ}W$ outcrops at point D on the north side of the fault and point E on the south side of the fault. A third vein with a dip of $20^{\circ}S70^{\circ}W$ outcrops at point F.*

*Point B is 300 feet north of A
Point C is 300 feet south of A
Point D is 2500 feet east of B
Point E is 2500 feet east of C
Point F is 1500 feet east of E*

Find the true displacement of the fault and find continuation of third vein on north side of fault.

The vertical fault and the six points are drawn in the plan view as shown in Figure 14. Through points B, C, D, E, and F strike lines are drawn for the veins and extended to intersect the vertical fault. The direction of dip is indicated on each strike line.

Next draw views 2, 3, and 4 with the folding lines perpendicular to the strike. The angle of dip is laid off in the proper direction in each view and an auxillary plane "h" distance below the folding line is drawn. This h distance is the same wherever used in the solution of this problem. It represents the elevation of a structure contour line on the vein at h distance below the surface. Points M and L are the location projected into the plan view of the intersection of these structure contours and the vertical fault.

Next it is necessary to find the line of intersection of the veins on the fault. To do this the fault is rotated into the horizontal or plan view about its trace at the surface. Since this is a vertical fault the structure contour at h elevation on the fault when rotated into the horizontal will be h distance from the trace and is drawn as RR on Figure 14. Points M and L which are the location of the intersection of the structure contours of the veins and the fault must also be rotated into the horizontal. This is done by drawing perpendiculars from M and L to RR. Lines are drawn from J through the intersection of M on RR and K through the intersection of L on RR to their intersection at S. These lines are the trace rotated into the horizontal of the veins on the south wall of the fault and S is their point of intersection. Lines parallel to JS and KS are drawn from the intersection of the veins on the north side of the fault and the fault to their intersection at N. These lines are the trace of the veins on the

north wall of the fault and N is their intersection. Since S and N were together before faulting line SN is the net slip. The dip and strike components can be determined by constructing the horizontal and vertical components as shown.

Point N is down and to the east of point S so the relative movement of the fault is the north block moved down and to the east in relation to the south block. The net slip is 330 feet, the strike slip is 90 feet, and the dip slip is 310 feet.

To find the extension of the third vein on the north side of the fault, view 4 is drawn with angle of dip and h distance laid off and point P found by projection. A perpendicular from P to RR is made and the line from Q through the projection drawn. SN in its proper orientation and length is transposed to some convenient location such as S'N'. A line parallel to S'Q is drawn from N' to the vertical fault. This is the point where the vein on the north side of the fault intersects the fault. The strike of the vein is drawn and point G on the vein is found to be 2,280 feet east of point D.

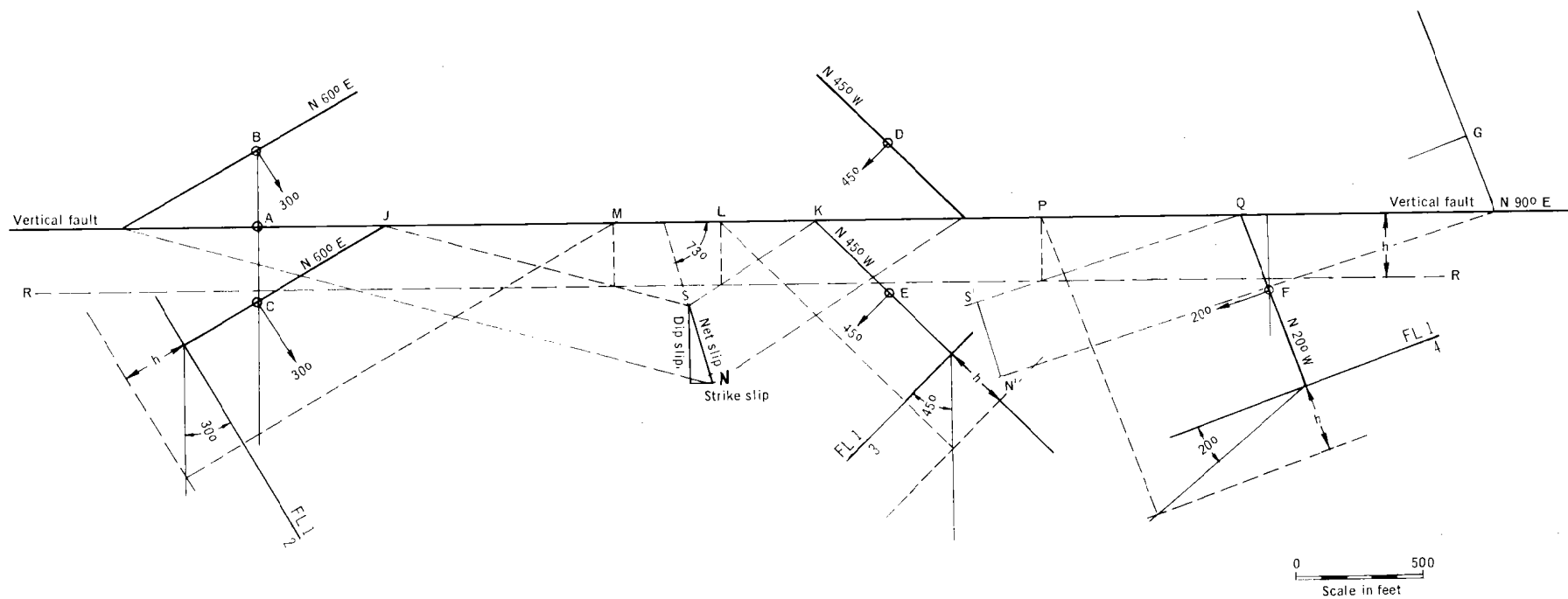


FIGURE 14. - (repeated)

Displacement of an Inclined Fault

The graphical solution of an inclined fault problem is much the same as with the vertical fault. *Figure 15 is the solution of the same problem as given for Figure 14 except in this problem the fault dips 45° south.*

The plan view is laid out and views 2, 3, and 4 drawn as before. The distance h , an arbitrary distance below the folding lines, defines an auxiliary plane or structure contour and h remains constant wherever used throughout the problem.

View 5 of the fault is drawn, the angle of the fault (45°) and the h distance plotted. Line RR is the trace on the fault of the structure contour h distance below the surface projected into the plan view. Points M, L, and P are the projection into the plan view of the intersection of the structure contours on the veins with the structure contour on the fault.

To find the net slip of the fault, the fault must be rotated about VV into the horizontal or plan view. This is done by swinging an arc in view 5 using the intersection of VV and FL 1/5 as the center and the intersection of h and the dipping fault as the radius. Line TT is drawn parallel to VV through the point where the arc intersects FL 1/5.

Perpendiculars are dropped from points M and P to TT. The point S' is defined by the intersection of lines from J through the projection of M on TT and K through the projection of L on TT. N' is defined by drawing lines from the intersection of the veins on the north side of the fault with VV parallel to JS' and LS'. S'N' is the net slip of the fault (1050 feet).

The projection into the plan view of the net slip of the fault is SN. This is found by drawing lines JM and KL to S and lines parallel to JS and KS from the veins on the north side of the fault to N. The relative movement along the fault is the north side moved down and to the east in relation to the south side.

To find the plunge of the net slip, a view parallel to NS to show NS in its true position can be constructed. This view can be moved to an uncluttered part of the paper and constructed in the following steps. Find the difference in elevation between points N and S. This is accomplished by projecting points N and S parallel to VV to their intersection with the fault in view 5. Since N and S were together before faulting, projecting their intersection on the fault to line VV gives the interval 1, 2, which is their difference in elevation. On a separate part of the paper lay off the distance (850 feet) SN. From N drop a perpendicular equal to the difference in elevation 1-2, then draw S-2. The plunge of the net slip is 44° , the net slip (S-2) is 1050 feet and is equal to S'N'.

To find the location of the third vein on the north side of the fault, a line is drawn from Q through the projection of P on TT. This line intersects S'N' at S'. A line parallel to QS' is drawn from N' to VV. The strike of the third vein is drawn from this intersection on VV. Point G on the third vein is found 3,620 feet east of point D.

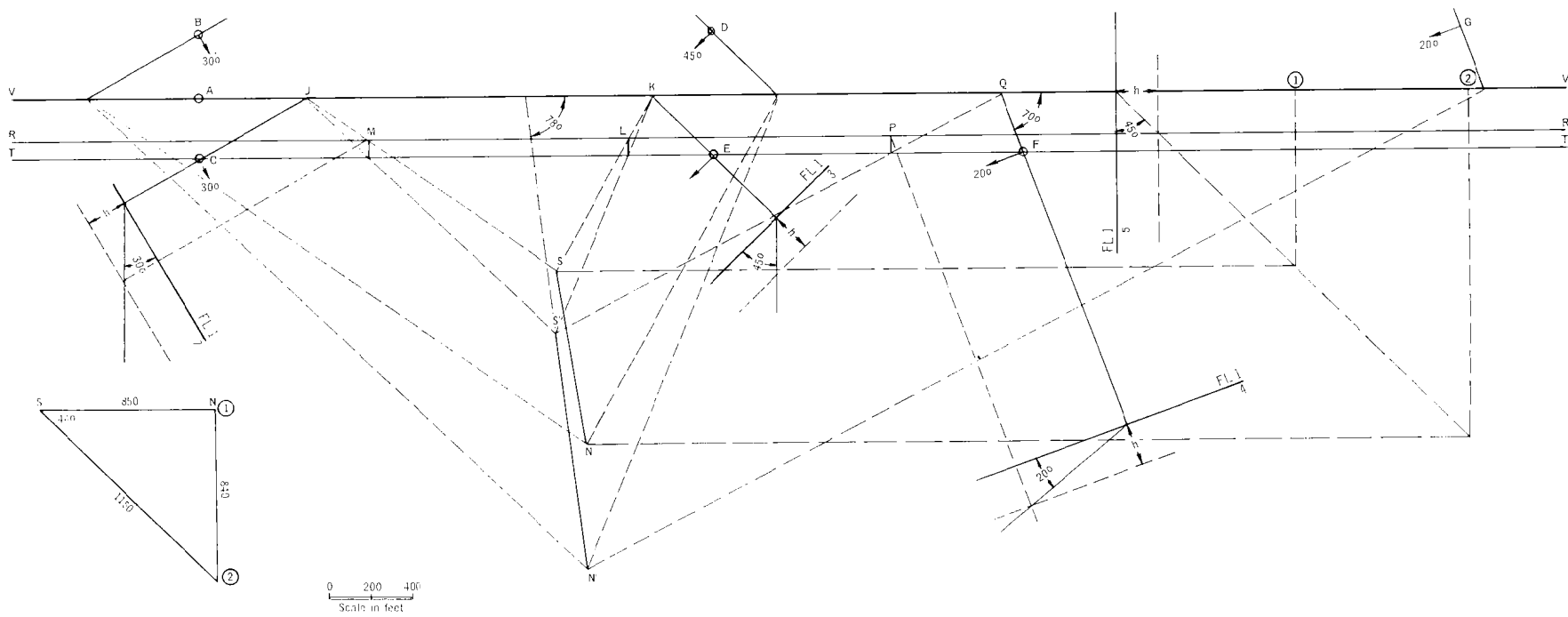


FIGURE 15. --Displacement of an inclined fault

Stereographic Projection

Stereographic projection is a rapid method of solving some geologic problems if angles and spatial relations between lines and planes are needed. The following examples illustrate some of the uses of stereographic projections.

The stereographic or Wulff meridional stereonet is shown in Figure 16. Extra copies are provided at the back of this technical release. If a sphere with meridional or great circles and pole or small circles drawn two degrees apart on its surface was cut in half through the poles, Figure 16 is a projection of these arcs on the equatorial plane. The bearing of lines or planes is measured from the north and south poles along the small circles. The dip of lines and planes is measured along the great circles, the amount (degrees) of dip being counted in from the periphery of the net along the east-west axis.

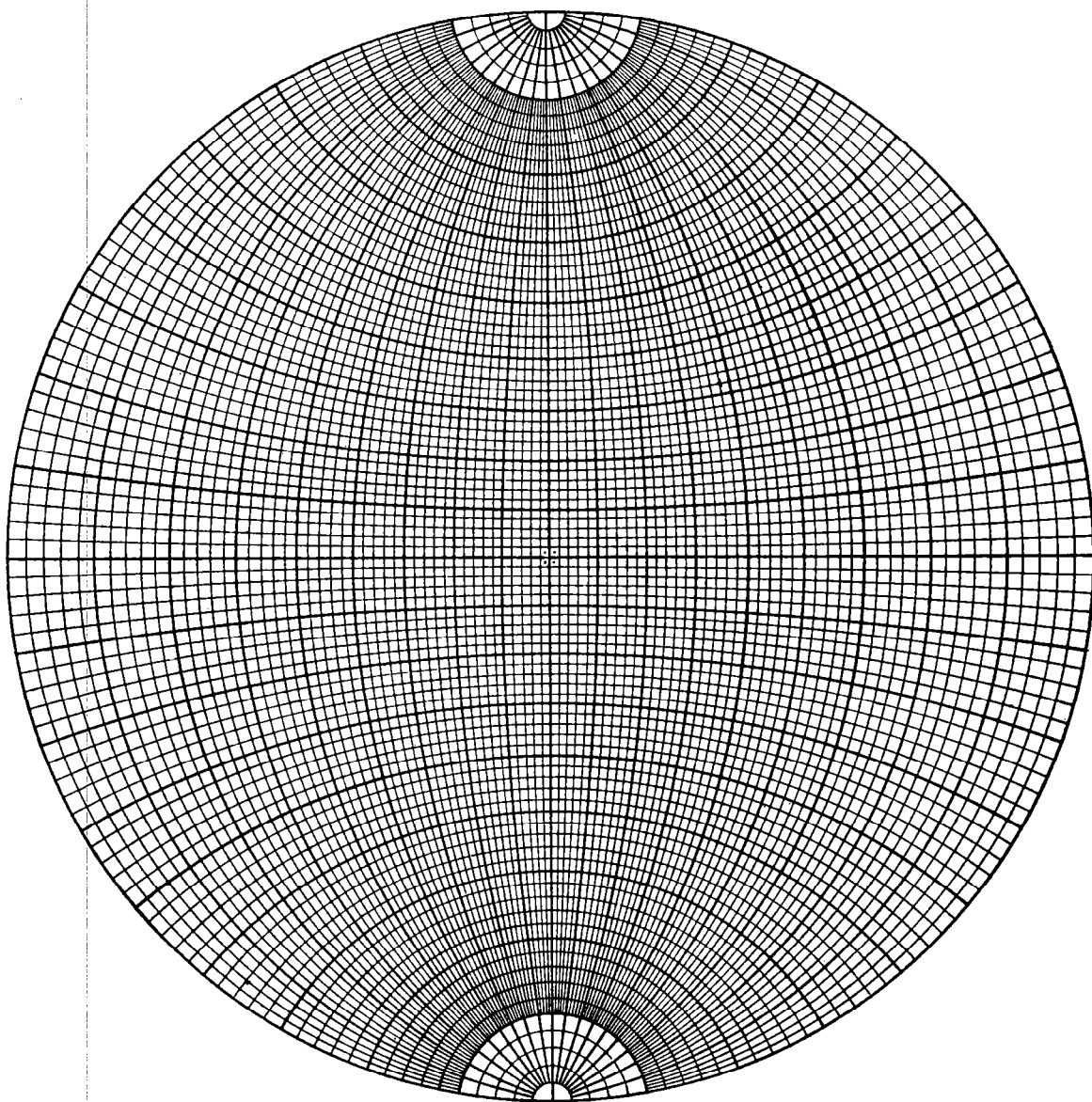


FIGURE 16. - Wulff net

True Dip from Two Apparent Dips

If, from a common point, two apparent dips are measured, $30^{\circ}\text{N}40^{\circ}\text{E}$ and $15^{\circ}\text{N}15^{\circ}\text{E}$, determine the bearing and amount of true dip.

Figure 17 is the solution of this problem. The stereonet, Figure 16, is taped to a desk or drawing board and overlaid by tracing paper. The tracing paper is fastened at the center of the net either by a pin or a reversed thumb tack placed beneath the net so that it may be rotated. The north, south, east, and west points on the perimeter of the net are marked on the tracing paper.

With the four cardinal compass points marked on the tracing paper and in their true positions with respect to the stereonet, lines indicating the bearing of the two apparent dips ($\text{N}15^{\circ}\text{E}$ and $\text{N}40^{\circ}\text{E}$) are drawn from the center of the net to the edge. Next the paper is rotated so the $\text{N}40^{\circ}\text{E}$ line coincides with the east line of the net. The amount of dip (30°) is counted in from the perimeter and marked. The $\text{N}15^{\circ}\text{E}$ line is then rotated to the east diameter and 15° counted in from the perimeter and marked. The paper is then rotated until the two apparent dips (15° and 30°) lie on the same great circle. The great circle is traced and the north-south (strike) and east (dip) diameter drawn. The amount of true dip is 40° counted in from perimeter of great circle on the east diameter. The paper is then rotated to its original position with the two apparent dips in the $\text{N}15^{\circ}\text{E}$ and $\text{N}40^{\circ}\text{E}$ direction and the bearing of true dip is read as $\text{N}86^{\circ}\text{E}$.

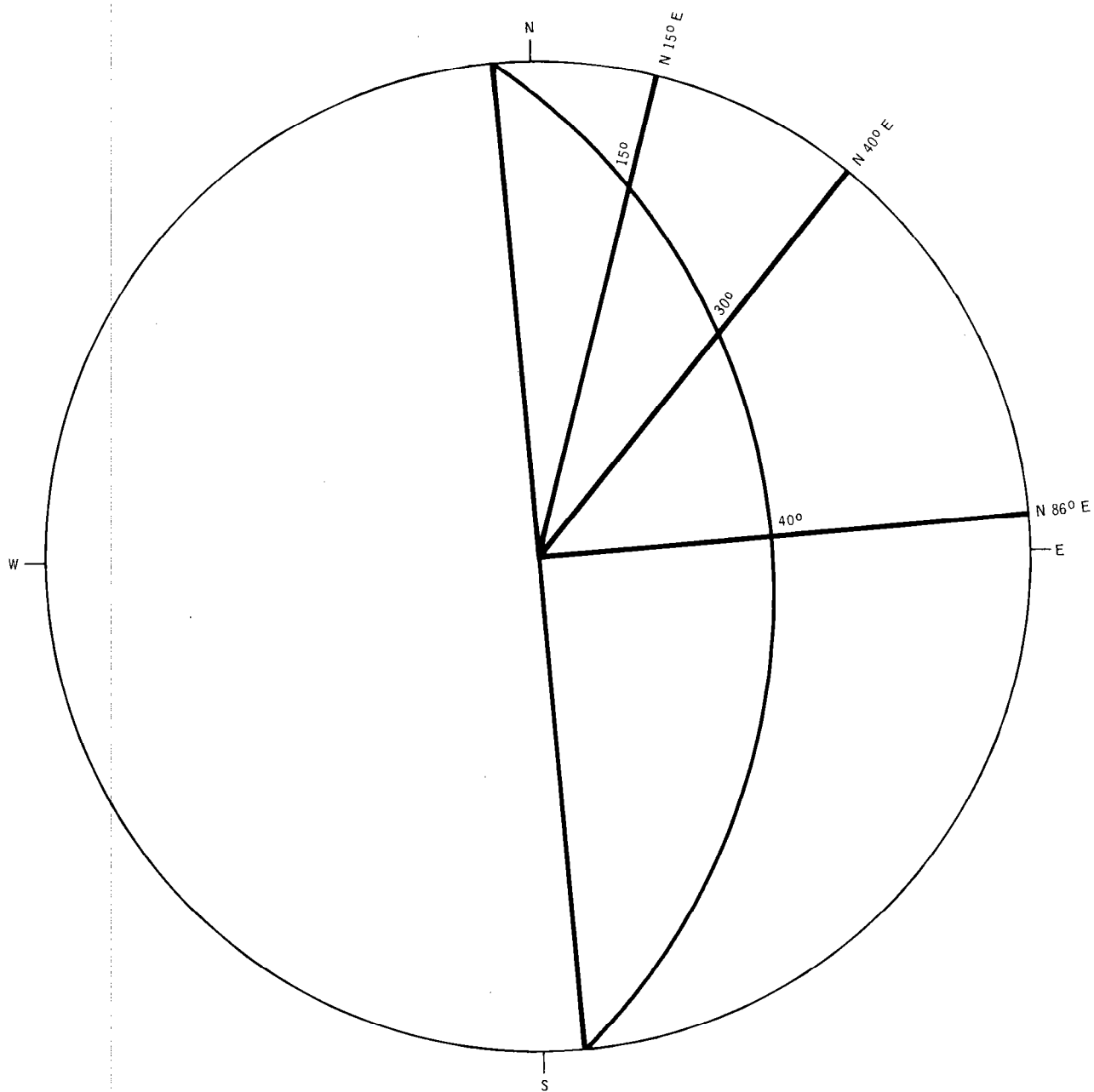


FIGURE 17.—True dip from two apparent dips.

Apparent Dip from True Dip

When drawing cross sections and other type illustrations, it is not always possible to draw them perpendicular to the strike. In these cases the apparent dip should be plotted, not true dip. The stereonet is a fast method of obtaining apparent dips when the true dip is known.

The following problem is an example. *A bed dips $30^{\circ}\text{N}40^{\circ}\text{W}$. What is the apparent dip in the $\text{S}70^{\circ}\text{W}$ direction?*

Figure 18 is the solution of this problem. The bearing of true dip ($\text{N}40^{\circ}\text{W}$) and direction of apparent dip ($\text{S}70^{\circ}\text{W}$) are plotted; the bearing of true dip is rotated to the west diameter; 30° counted in from the perimeter; and the great circle is drawn. The paper is then rotated so the line of the bearing of the apparent dip desired ($\text{S}70^{\circ}\text{W}$) is on the west diameter and the amount of apparent dip (11°) counted in from the perimeter.

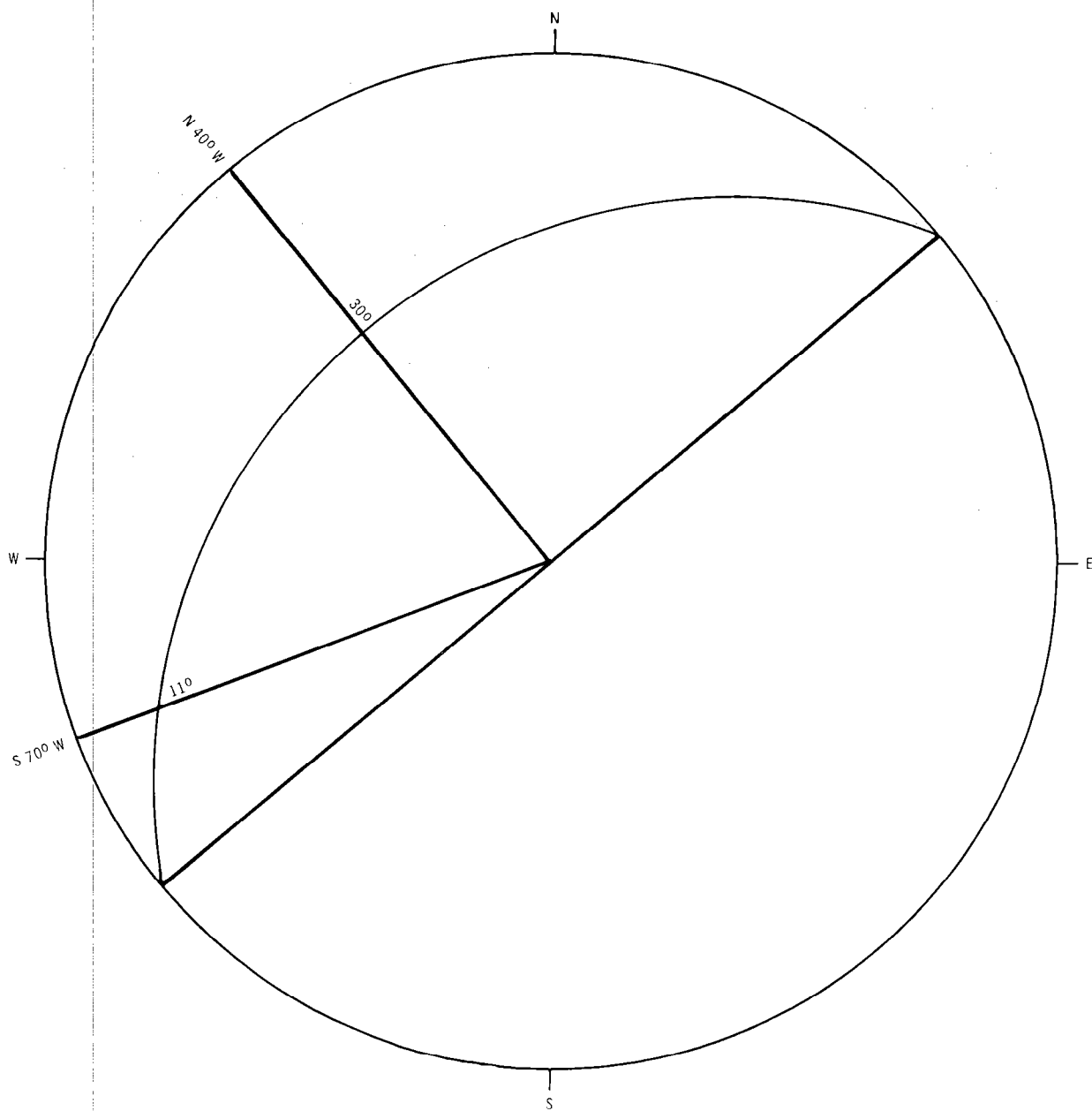


FIGURE 18 – Apparent dip from true dip

Line of Intersection of Two Oblique Planes

A bed dips $30^{\circ}\text{N}10^{\circ}\text{W}$ and a fault dips $20^{\circ}\text{S}45^{\circ}\text{W}$. What is the bearing and plunge the line of intersection between the bed and the fault and what is the rake of this line in the plane of the bed and in the plane of the fault?

Figure 19 is the solution. The bearing of the dip of the bed and the fault are drawn. The $\text{N}10^{\circ}\text{W}$ line is rotated to the west diameter and 30° counted in and the great circle drawn. This is repeated for the $\text{S}45^{\circ}\text{W}$ line. A line from the center of the net through the point of intersection of the two great circles is the line of intersection of the two planes. Rotating the tracing paper to its original position the bearing of the intersection is $\text{N}81^{\circ}\text{W}$. Rotate the paper so the $\text{N}81^{\circ}\text{W}$ line is on the west diameter and the plunge of the intersection counting in from the periphery is 12° .

To determine the rake of the line of intersection in the plane of the fault rotate the tracing paper so the strike of the fault is along the north-south axis. The rake (36°) is found by counting the small circles from the north pole along the great circle of the fault to the point of intersection determined above. To determine the rake of the line for the bed, the strike of the bed is placed on the north-south axis and the angle found by again counting the small circle to the point of intersection.

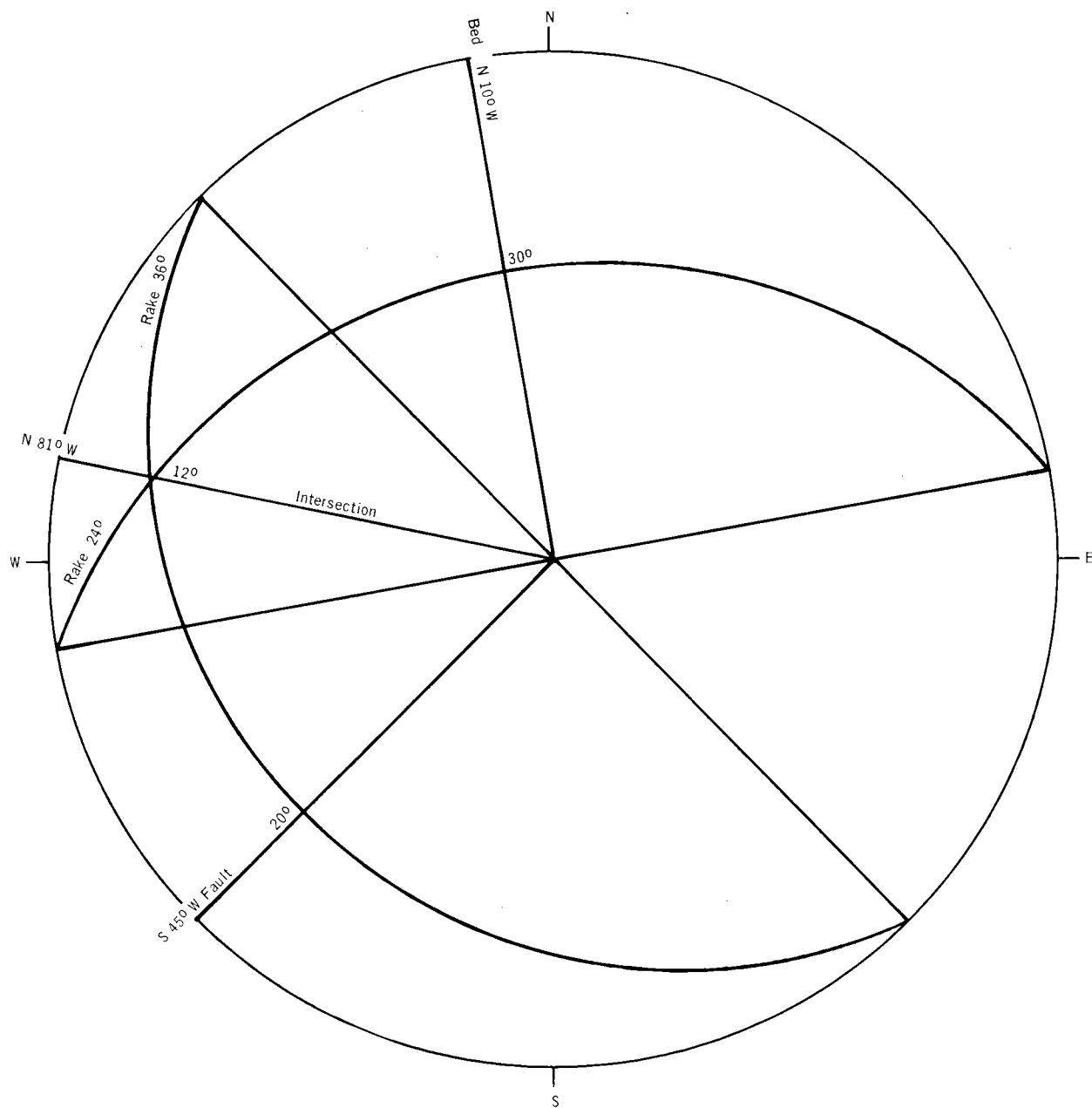


FIGURE 19 – Intersection of two oblique planes

Rotation of a Bed

Occasionally it is desirable to restore to their original position beds, faults, and joints that have been rotated. This is easily and quickly done by stereographic projection. The following problem is an example.

Two beds are separated by an unconformity. The top bed dips $15^{\circ}\text{S}20^{\circ}\text{E}$ and the lower bed dips $40^{\circ}\text{N}20^{\circ}\text{E}$. Find the dip and strike of the lower bed when the top bed was horizontal (being deposited).

In Figure 20 the great circles for the dip and strike of the two beds are drawn as in the previous examples. To find the dip and strike of the lower bed when the top bed was horizontal it is necessary to rotate the top bed into the horizontal and the bottom bed through the same amount of rotation. This is accomplished by moving the paper so the strike line of the top bed is on the north-south diameter. All points on the great circle of the top bed when it is rotated 15° into the horizontal will fall on the perimeter of the net. This includes point A which is the intersection of the two great circles. Likewise all other points on the great circle of the lower bed will also rotate 15° along the small circles. A few of these points are indicated by the dashed lines. Point A is a position of zero dip, so it is rotated to the north pole. A great circle is drawn from point A and connecting the ends of the dashed arcs. This great circle (dashed) represents the position of the lower bed ($52^{\circ}\text{N}12^{\circ}\text{E}$) when the top bed was horizontal.

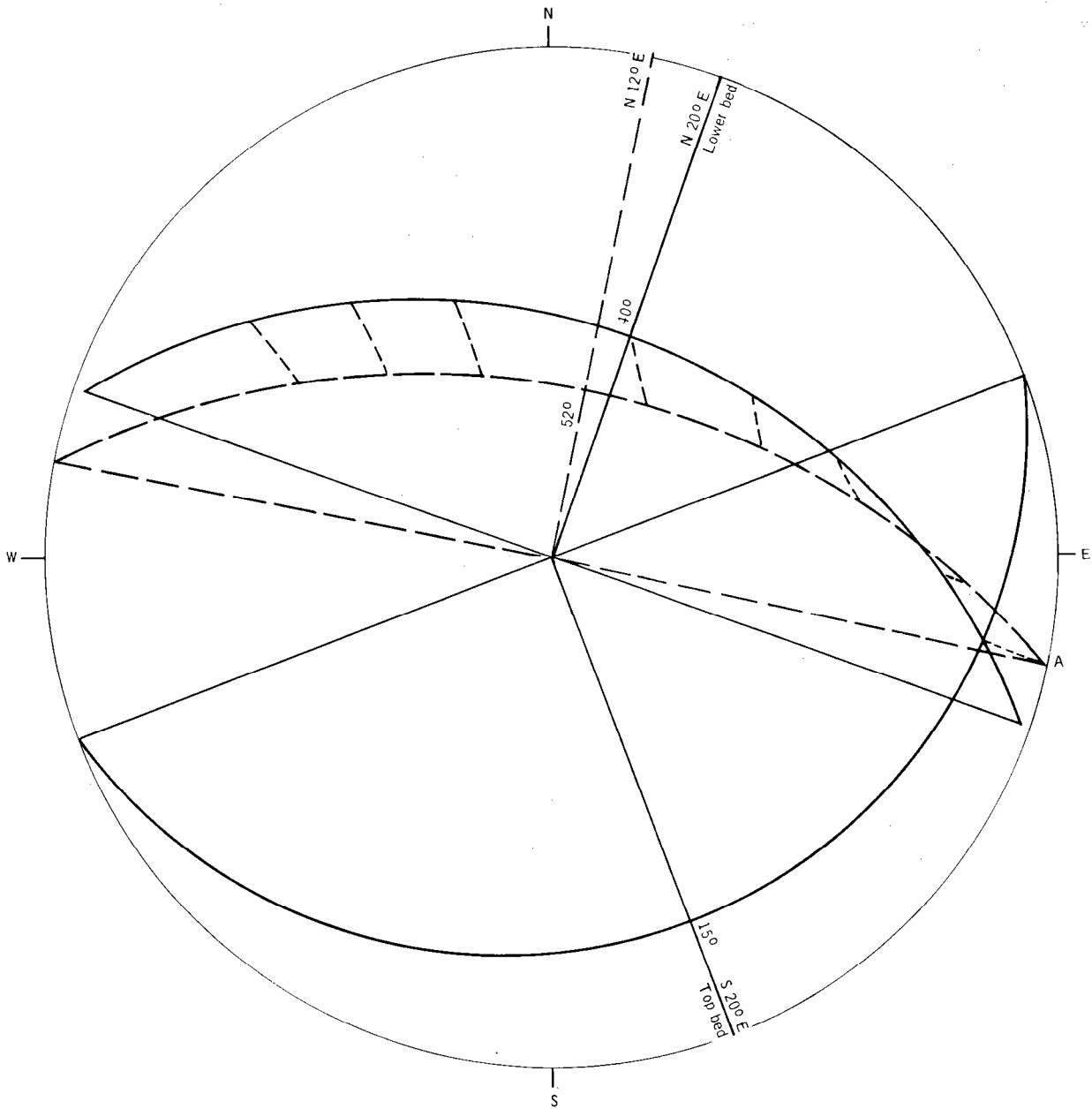


FIGURE 20. — Rotation.

Rotation of a Fault

If the same bed has a different dip and strike on opposite sides of a fault, rotation along the fault has occurred. In the previous example rotation of the bed was about a horizontal axis. To solve problems of rotation about an inclined axis, an additional step of rotating the axis into a horizontal or vertical position before rotating the beds is required.

The following problem is an example. *A fault dips $30^{\circ}\text{N}20^{\circ}\text{E}$. A bed in the south block dips $10^{\circ}\text{N}20^{\circ}\text{W}$, and a bed in the north block dips $28^{\circ}\text{N}30^{\circ}\text{W}$. What has been the rotation of the north block with respect to the south block (angle and clockwise or counter-clockwise) and is the fault movement simple rotation?*

The problem can be solved in three steps. First the fault plane is rotated into the horizontal, the beds are moved through the same angle of rotation, the angle of rotation between the beds can then be measured. Second the fault plane is rotated to the vertical and the beds again move the same amount. Third the north bed is rotated through the angle of rotation determined in the first step about an axis perpendicular to the fault plane to see if it coincides with the south bed. If it does, the fault movement is simple rotation.

In Figure 21A the great circles representing the attitude and dip of the two beds and the fault are plotted as before. Rotation has occurred along the fault, therefore, the axis of rotation of the fault must be perpendicular to the fault. To measure the angular difference (angle of rotation) between the two beds the axis of rotation of the fault is rotated to the vertical (fault rotated to horizontal). The two beds are rotated through the same angle. To do this the strike of the fault is placed on the north-south diameter and the dip on the east diameter. When the fault is rotated 30° into the horizontal its great circle coincides with the periphery of the net. With the tracing paper held in the same position the two great circles representing the two beds are rotated 30° in the same direction as indicated by the dashed lines. Great circles, indicated by the hachure lines, are found by rotating the tracing paper until the points projected by dotted lines lie on the same great circle. The angle between the two beds can be measured, as indicated, on the periphery of the net.

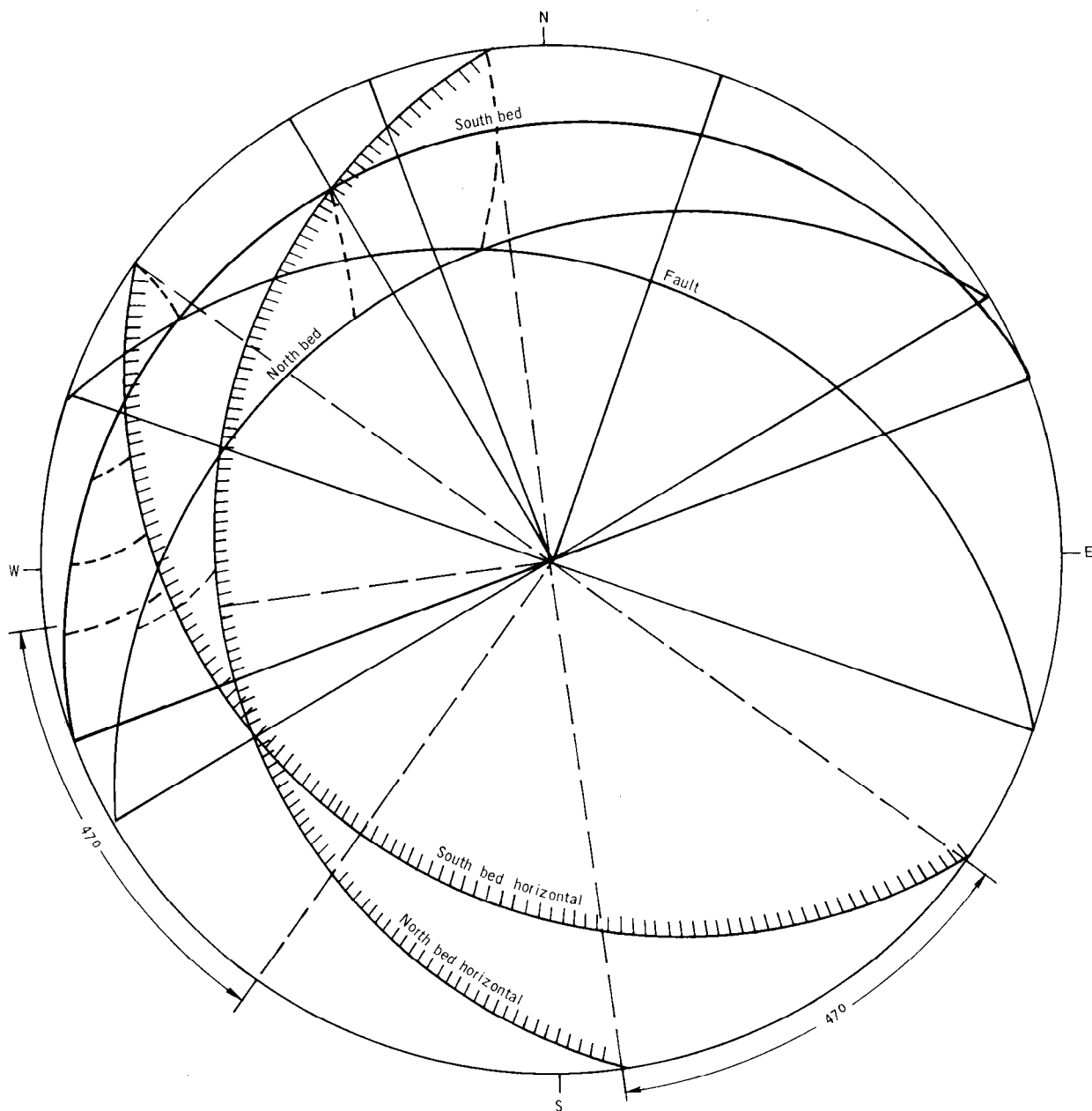


FIGURE 21 A.—Rotation about an inclined axis.

Figure 21B shows the second and third steps. In the second step the fault is rotated to a vertical position, the axis of rotation of the fault will then be horizontal and the beds can then be rotated about the horizontal axis. Rotation of the fault to the vertical is opposite of the horizontal rotation done in the first step. The strike of the fault (original plotted position again) is placed on the north-south axis and dip on the east diameter. To rotate the fault to the vertical it is rotated 60° down (east to west), the great circle will then coincide with the north-south axis of the net. With the tracing paper held in the same position the two beds are rotated 60° in the same direction. Their new locations are indicated by the hachured great circles.

The third step involves rotating the north bed through the angle determined in the first step about the axis of rotation normal to the fault. To do this the tracing paper is rotated so the strike of the fault is placed on the east-west diameter. The north bed is rotated 47° as indicated by the dotted lines.

When rotated through the angle of 47° the north bed coincides with the south bed, therefore, the movement of the fault has been simple rotation of 47° of the north block counter-clockwise with respect to the south block.

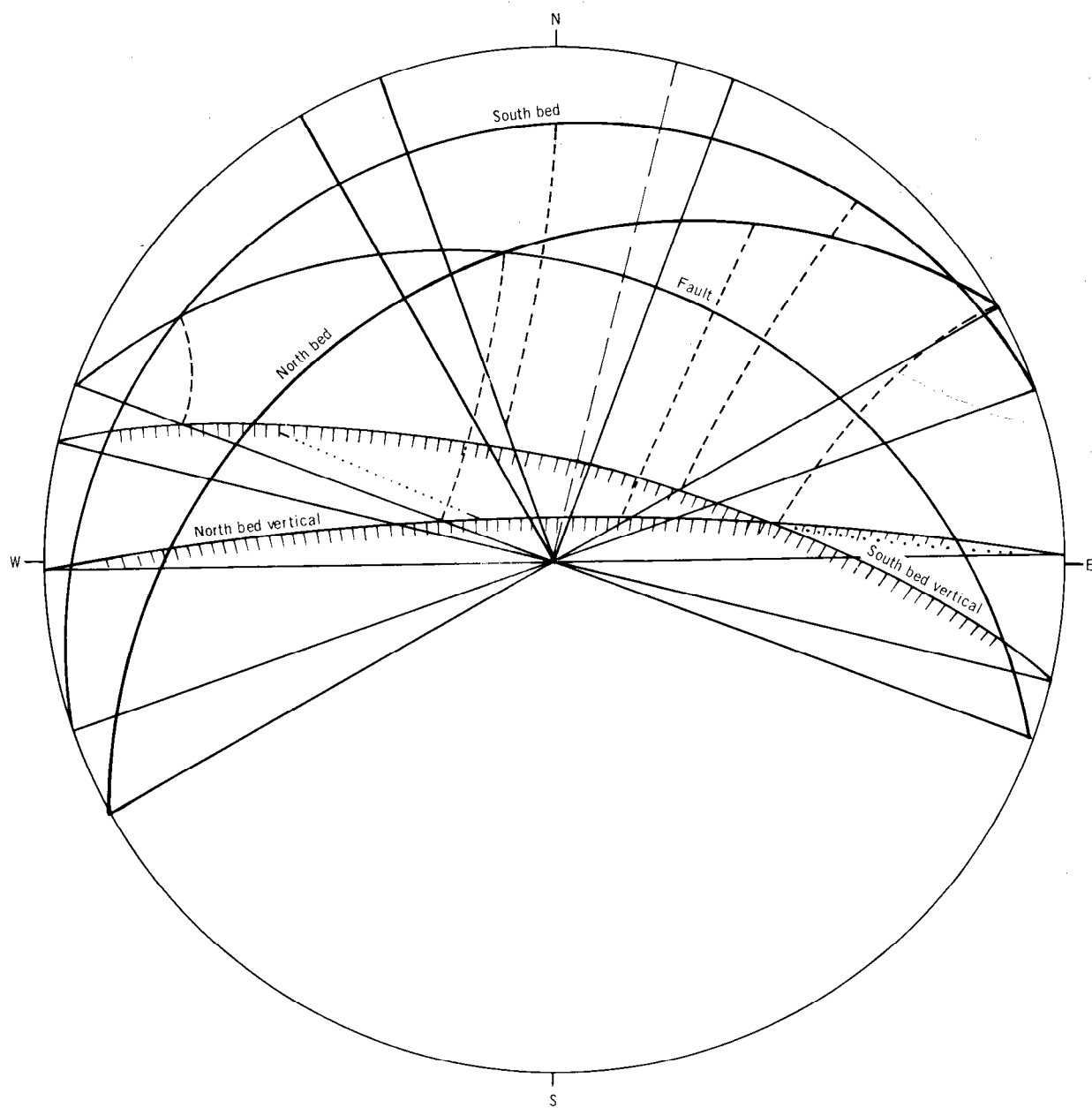


FIGURE 21 B

Poles

Rotation of beds on the stereonet can be made more expeditiously by using points representing the poles of planes instead of the planes themselves. The pole of a plane is a line perpendicular to the plane and passing through the center of the stereonet. Every plane, represented by a great circle on the stereonet, has a unique point also on the net that represents the pole of the plane.

Figure 22 illustrates the relationship between poles and planes. Figure 22A is a three dimensional drawing of the lower reference hemisphere. A plane (rock stratum) defined by points N B S O dips 45° due east. Line AO perpendicular to the plane and passing through the center of the net is the pole. Projecting the pole and plane to the stereonet in Figure 21B the plane is defined by the great circle N C S and the pole by point P. If the plane had dipped 30° N 45° E the location of the point defining the pole would be 60° in from the periphery or 30° out from the center of the net and have a S 45° W bearing.

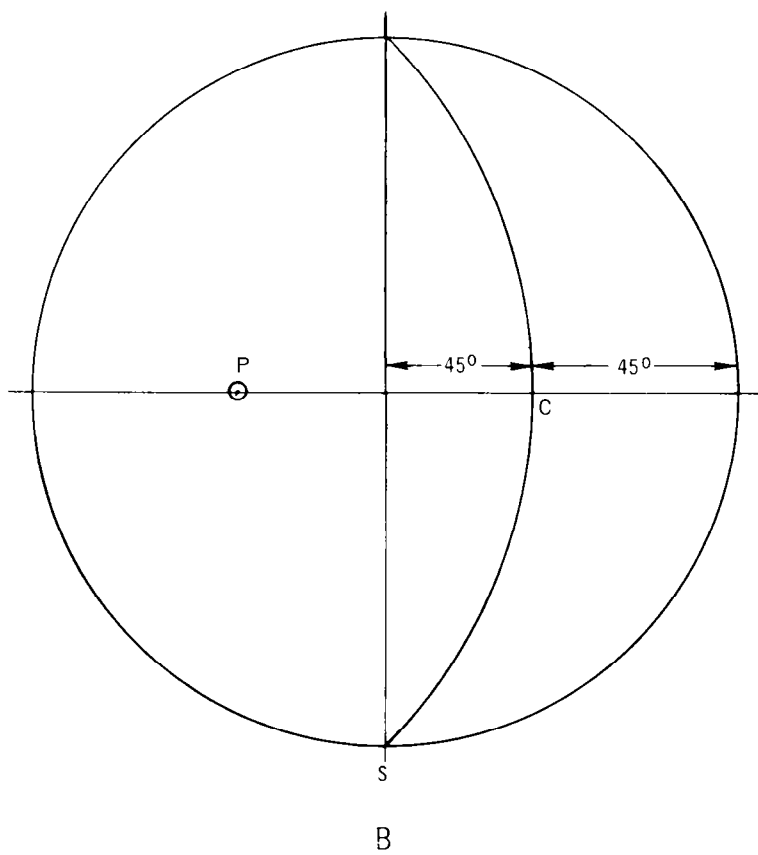
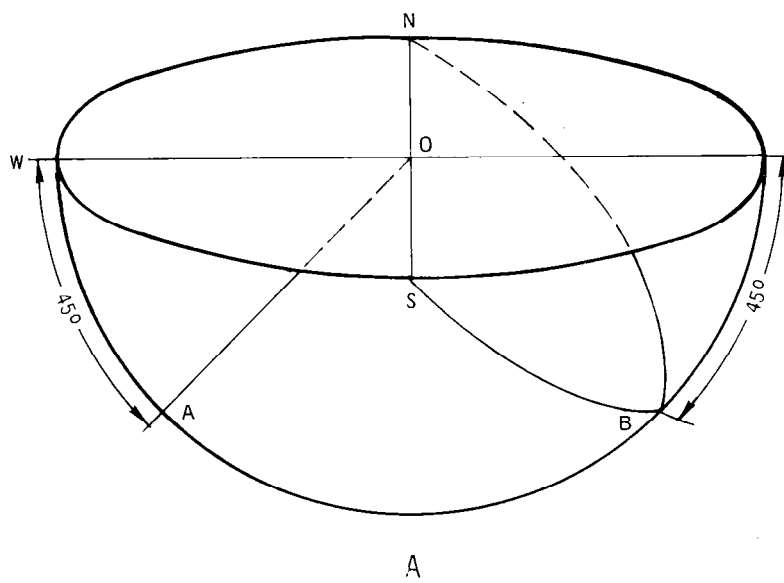


FIGURE 22.—Relationship between planes and poles.

Rotation of a Bed

The problem illustrated in Figure 20 can be solved using poles instead of planes. The problem reiterated is: *Two beds are separated by an unconformity. The top bed dips $15^{\circ}S20^{\circ}E$ and the lower bed dips $40^{\circ}N20^{\circ}E$. Find the dip and strike of the lower bed when the top bed was horizontal.*

The pole for the top bed is located 15° out from the center of the net (75° in from periphery) with a bearing of $N20^{\circ}W$. The pole for the lower bed is 40° out from the center of the net (50° in from periphery) with a bearing of $S20^{\circ}W$. Bearings are laid off on the perimeter of the net and dip counted off on the great circles on the east-west diameter and the location of the two poles are plotted. Point T is the pole of the top bed and point L the lower bed. To determine the attitude of the lower bed when the top bed was horizontal the top bed must be rotated into the horizontal. To accomplish this the tracing paper is rotated until point T is on the east-west diameter (west side of center). When the bed is rotated into the horizontal the pole will be vertical; therefore, point T moves 15° to 0 and, without moving the tracing paper, point L is rotated 15° along the small circle in the same direction at T to point L_1 . The point L_1 is the pole of the lower bed when the top bed was horizontal. The bearing of point L_1 is $S12^{\circ}W$ and the dip counted in from the periphery along the east-west diameter is 38° . The attitude of the lower bed, therefore, is 52° ($90^{\circ}-38^{\circ}$) $N12^{\circ}E$.

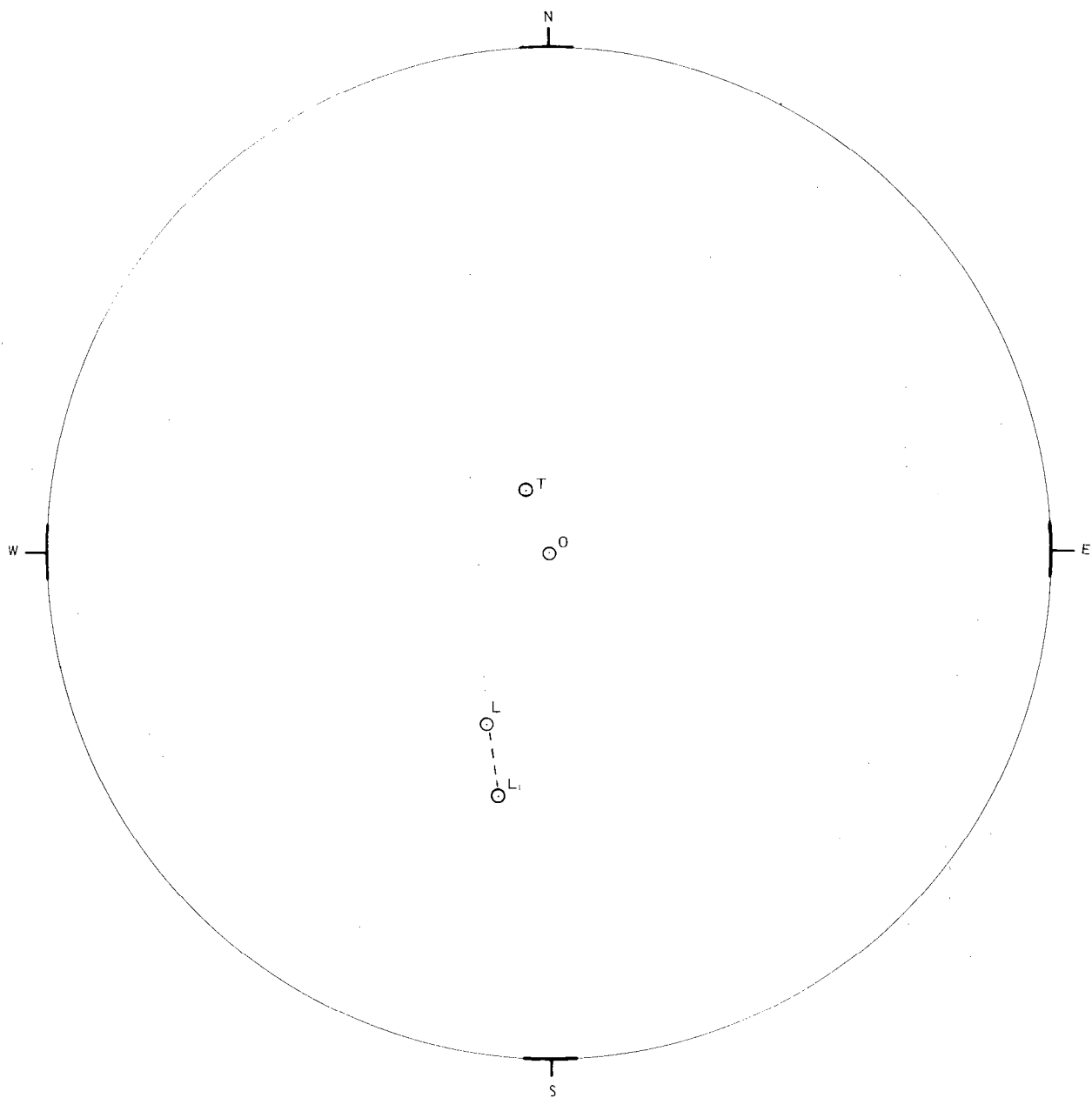


FIGURE 23. - Rotation using poles.

Rotation of a Fault

The problem solved in Figure 21 (rotation about a fault) can also be solved using points representing the poles of planes. Figure 24 is the solution using this method.

As given before, a fault dips $30^{\circ}\text{N}20^{\circ}\text{E}$, a bed in the south block dips $10^{\circ}\text{N}20^{\circ}\text{W}$, and a bed in the north block dips $28^{\circ}\text{N}30^{\circ}\text{E}$. What has been the rotation of the north block with respect to the south block and is the fault movement simple rotation.

The pole of the fault (PF) is $60^{\circ}\text{S}20^{\circ}\text{W}$ (all angles of dip counted in from periphery of net), the pole of the south bed (PSB) is $80^{\circ}\text{S}20^{\circ}\text{E}$, and the pole of the north bed (PNB) is $62^{\circ}\text{S}30^{\circ}\text{E}$.

The poles are plotted on the tracing paper as shown in Figure 24. To rotate the fault into the horizontal (axis of rotation vertical), the tracing paper is rotated until the pole of the fault (PF) is on the east-west diameter (west side). When the fault is rotated into the horizontal the pole (PF) will move 30° to the center of the net. Points PSB and PNB will also move 30° along their respective small circles as shown by dashed lines to points PSBH and PNBH. The angle of rotation of the fault is measured between lines from the center through points PSBH and PNBH. This angle can be conveniently counted along the small circles on the periphery as indicated.

To rotate the fault into the vertical point PF is again located on the west radius of the net and rotated out 60° or until point PF is on the periphery of the net. The points PSB and PNB are rotated along their respective small circles through the same 60° of rotation to points PSBV and PNBV.

To determine if simple rotation has occurred the north bed must be rotated 47° about the axis of rotation which is normal to the fault. This is accomplished by rotating the tracing paper until point PF is on the north radius of the net. Point PNBV is rotated 47° along its small circle where it coincides with point PSBV confirming the movement was simple rotation and that the north block was rotated 47° counter-clockwise with respect to the south block.

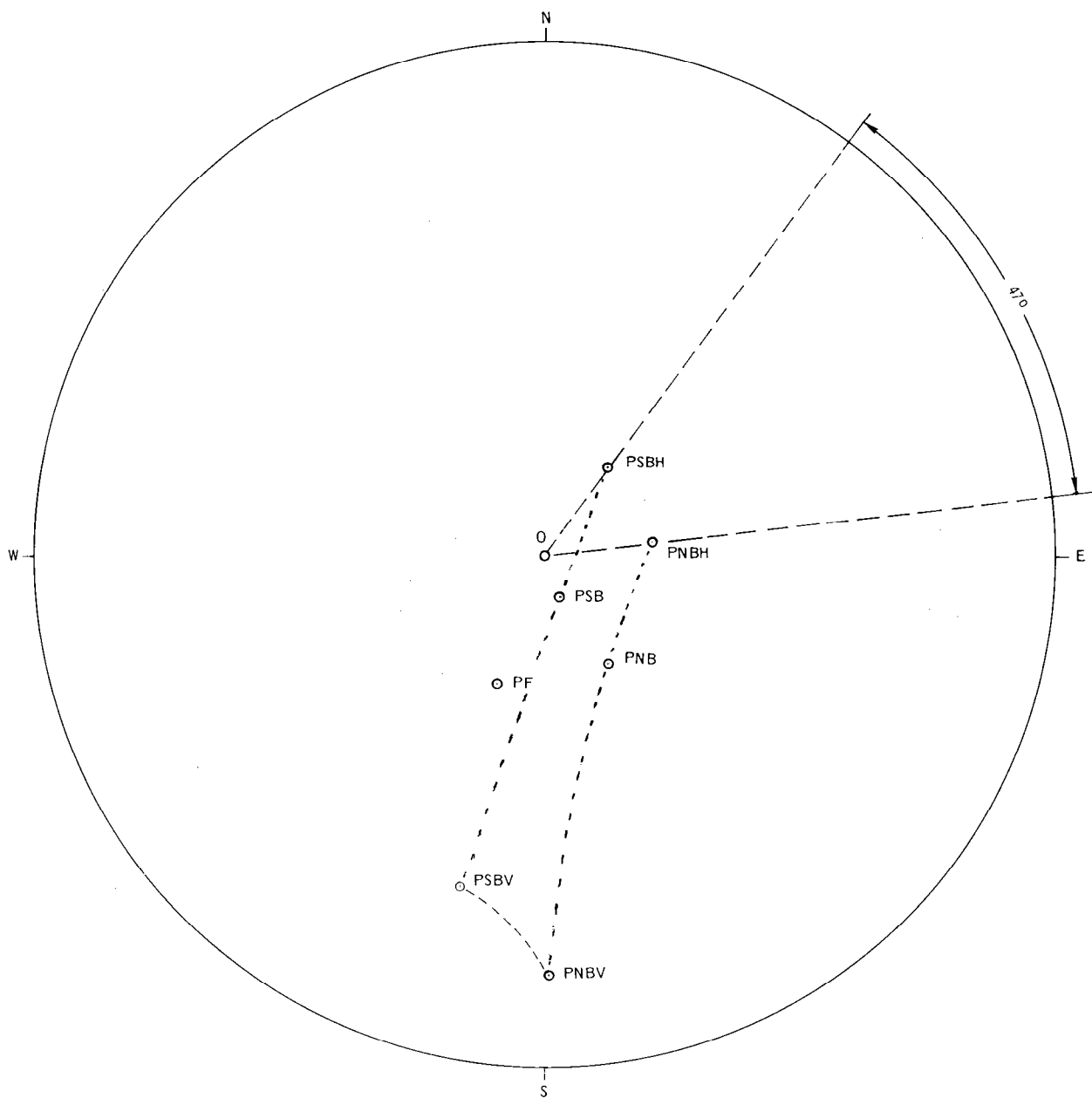


FIGURE 24 -Rotation about an inclined axis.

Vertical Drill Holes

The stereographic technique can be used to solve dip and strike problems involving unoriented cores from drill holes. The following is an example.

Points A and B are the locations of two vertical core test holes. The top of a key marker bed is encountered at elevation 157.5 in hole A and the core obtained shows that the bed dips at an angle of 45° . Since the core has been rotated in the core barrel, the direction of dip is unknown. Hole B is located 100 feet $N60^\circ W$ of hole A and the key marker bed was encountered at elevation 100.0. What is the attitude of the key marker bed?

From the information given (100 feet horizontally and 57.5 feet vertically) the apparent dip ($30^\circ N60^\circ W$) from A to B of the key marker bed can be determined either trigonometrically or graphically. With an apparent dip and bearing and the true dip known, two possibilities of the bearing of true dip can be found. More information, such as a third test core hole, is necessary to provide the unique solution of the bearing of true dip.

On the tracing paper overlying the stereonet plot the vector representing the direction and amount of dip from point A to B ($30^\circ N60^\circ W$). This is O-AB in Figure 25. Next rotate the tracing paper until the end of the vector (point AB) lies on a great circle representing 45° of dip. There are only two great circles of 45° dip that point AB will fall on as shown in Figure 25. Only one of these gives the bearing of true dip, but until more information is provided we cannot determine which one. The two possibilities of true dip are $45^\circ S68^\circ W$ and $45^\circ N7^\circ W$.

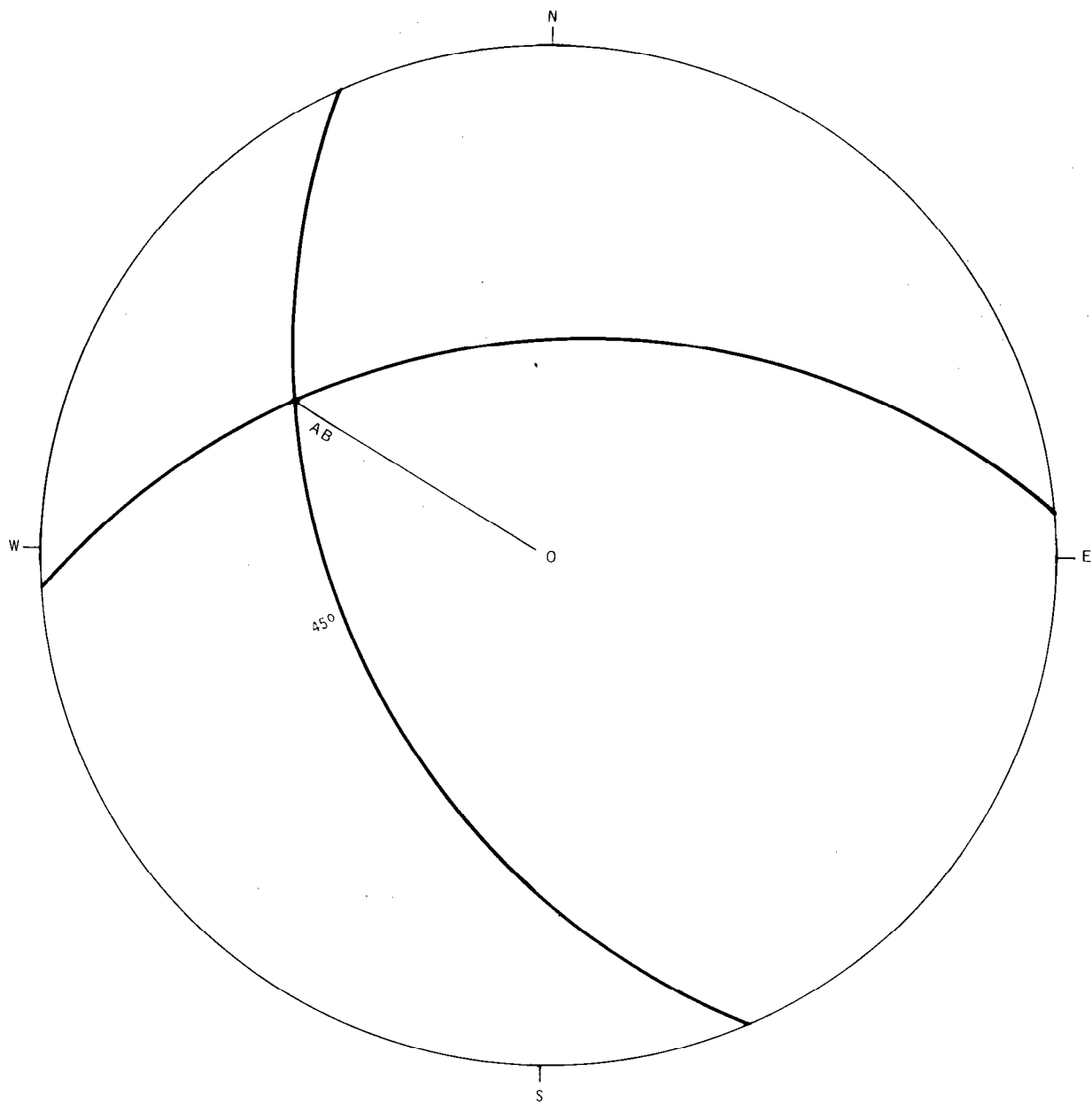


FIGURE 25 – True dip from vertical core holes.

Inclined Drill Hole

As another example, consider the same problem except only the elevation of the top of the bed at 157.5 in hole A is known and hole B is inclined from the vertical 40° in a $S60^\circ W$ direction (dips $50^\circ S60^\circ W$) and the beds make an angle of 45° with the core axis (dip 45°).

This is essentially the same problem as the previous one except it will also involve rotation. First plot the vectors representing direction and dip of the bed from A to B and the direction and dip of the drill hole at B. These are points AB and DH on Figure 26. Next, the inclined drill hole is rotated to the vertical. The tracing paper is rotated so that point DH is on the west radius. When DH is rotated to the vertical it moves to O and point AB moves through 40° to ABV. Next, as in the previous problem, rotate the tracing paper and draw the two 45° great circles through ABV. The last step is rotate the projection back to its original position. Place the original bearing of DH on the west radius and rotate DH from O to 40° . The two great circles through ABV will also rotate 40° to the positions indicated by the hachured great circle. These two great circles are the two possibilities of dip and strike of the key bed. When the tracing paper is rotated to its original position over the net, they are $42^\circ N11^\circ W$ and $70^\circ N18^\circ E$.

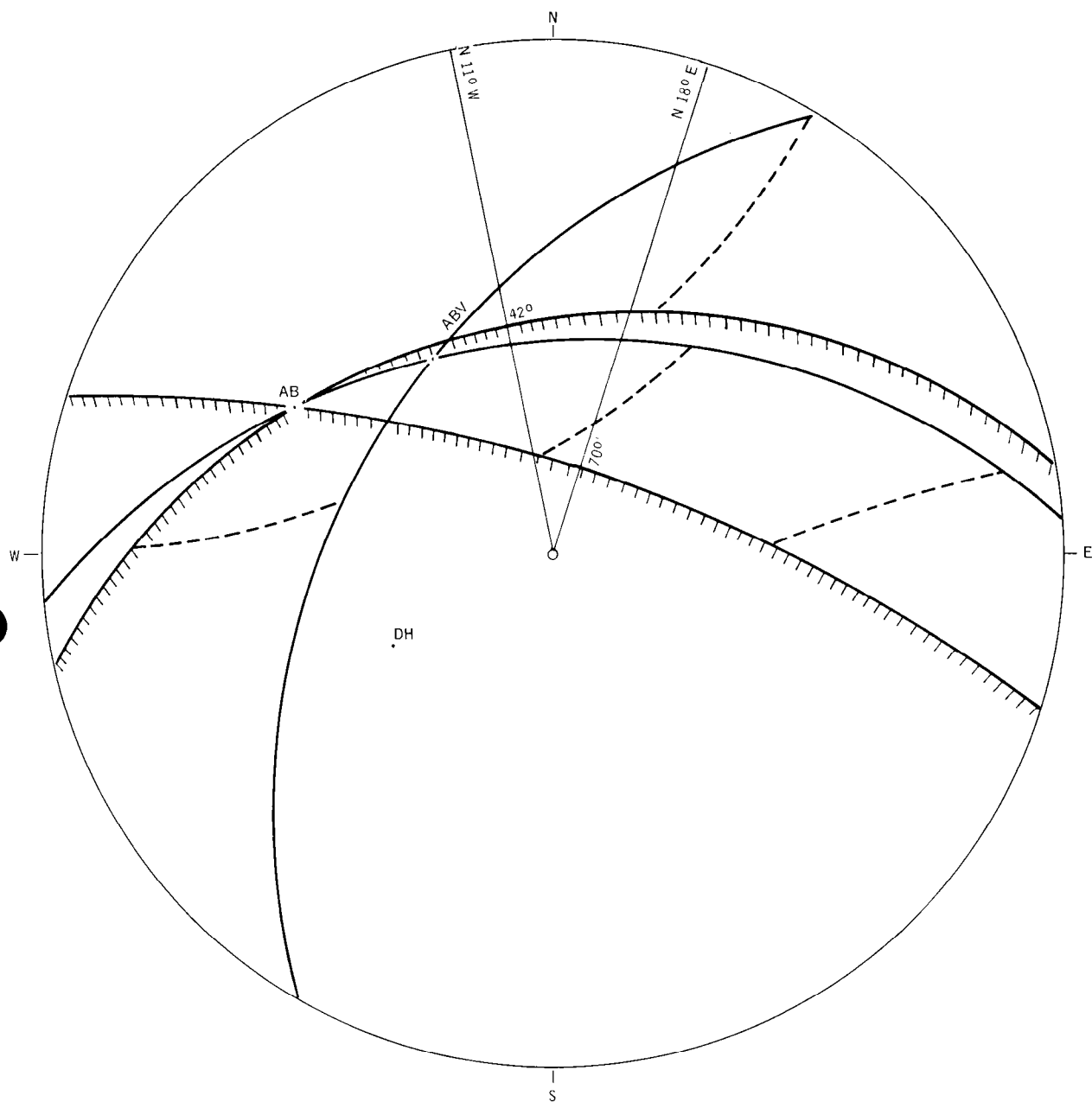


FIGURE 26.—True dip from an inclined core hole.

Combination Orthographic and Stereographic Technique

When solving problems involving displacement of non-rotational faults a combination of orthographic and stereographic procedures are oftentimes simpler to use than straight orthographic projections. The inclined fault problem in Figure 15 can be solved in the following manner by using this combination method.

Figure 27 is the plan view. The third vein at point F on the south side of the fault has not been shown.

The great circles representing the fault and the two veins are plotted on the stereonet in Figure 28. Great circle EXYW represents the fault, BXB¹ the vein located at points B and C, and DYD¹ the vein located at points D and E. Line XO on the stereonet is the horizontal projection of the trace of the intersection of the vein at B and C with fault. Likewise, line OY is the trace of the intersection of the vein at D and E with the fault.

To find the horizontal projection of the net slip, plot the bearings (from the stereonet) of line OX and OY on the plan view. Lines XS and X¹N are the bearing of OX ($90^\circ - 59^\circ = 31^\circ$), and YS and Y¹N are the bearing of OY ($90^\circ - 23^\circ = 67^\circ$). Their intersection is at S and N. Since points S and N were together before faulting, SN is the horizontal projection of the net slip (850 feet). The bearing of the net slip (SN) is S19°E. This bearing is plotted on the stereonet as O-SN. By rotating O-SN until SN is on the South pole and counting in on the small circles to the intersection of O-SN with the great circle of the fault the plunge of the net slip is 44°.

Next, rotate the tracing paper and place the strike of the fault on the north-south diameter. Counting the angle in on the small circles, EX (38°) is the rake of vein B in the plane of the fault and WY (74°) is the rake of vein D in the plane of the fault. The rake of vein B is southeast and vein D southwest.

To determine the total net slip, return to the plan view (Figure 27) and rotate the fault into the horizontal. Since points X¹, X, Y, and Y¹ are at the surface, they do not move when the fault is rotated. The angle of rake of a fault is measured in the plane of the fault (see Figure 12). If the angle of rake, as determined from the stereonet, of the veins on the fault are plotted in the plan view, the fault will be rotated into the horizontal. From points X¹ and X lay off the rake angle (38°) in a southeast direction, and at Y and Y¹ the rake angle (74°) in a southwest direction. The intersection of these lines at S¹ and N¹ determines line S¹N¹ which is the net slip (1150 feet) of the fault.

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